What else we have been doing and plan to do at OSU*

Ulrich Heinz (The Ohio State University)

JET Collaboration Meeting, UC Davis, June 17-18, 2014

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Overview

- Matching (free-streaming) pre-equilibrium dynamics to viscous hydro and studying sensitivity of observables to matching ("thermalization") time **(Liu Jia)**.

- HBT correlation afterburner. HBT interferometry for fluctuating sources **(Christopher Plumberg)**.

- Viscous anisotropic hydrodynamics **(Dennis Bazow)**

Pre-equilibrium dynamics (I)

Match pre-equilibrium $T^{\mu\nu}$ to **viscous** hydrodynamic form, at varying matching times τ_{match}.

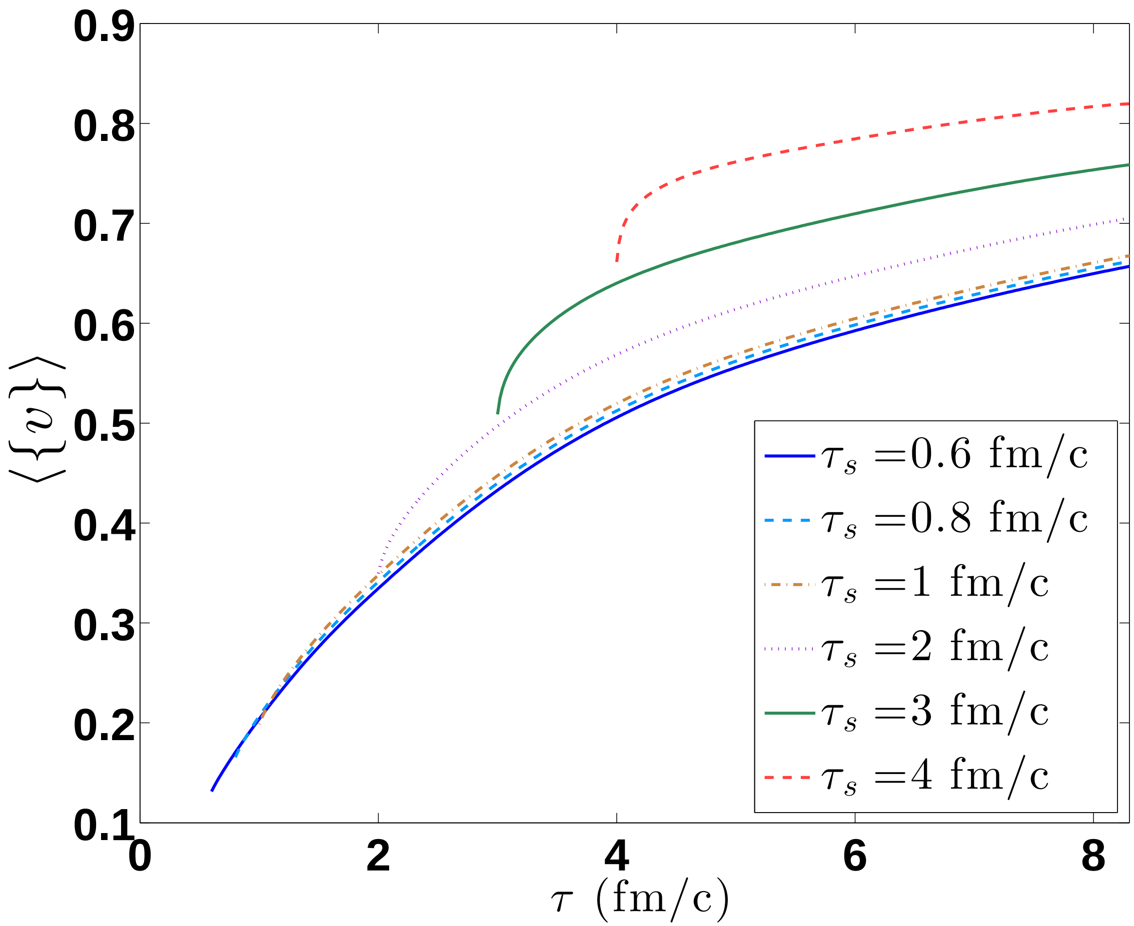
Extreme case: pre-equilibrium $=$ free-streaming
$\Longrightarrow$ large $\tau_{\mathrm{match}} \leftrightarrow$ slow thermalization; short $\tau_{\mathrm{match}} \leftrightarrow$ fast thermalization.

Study dependence of final observables on τ_{match} and compare with pure hydro calculation that assumes **no evolution at all** between $\tau = 0$ and $\tau_{\mathrm{therm}} = 0.7\,\mathrm{fm}/c$.

The following study by **Jia Liu** uses MC-KLN initial conditions for the gluon phase-space distribution. Viscous hydro evolution with $\eta/s = 0.2$.

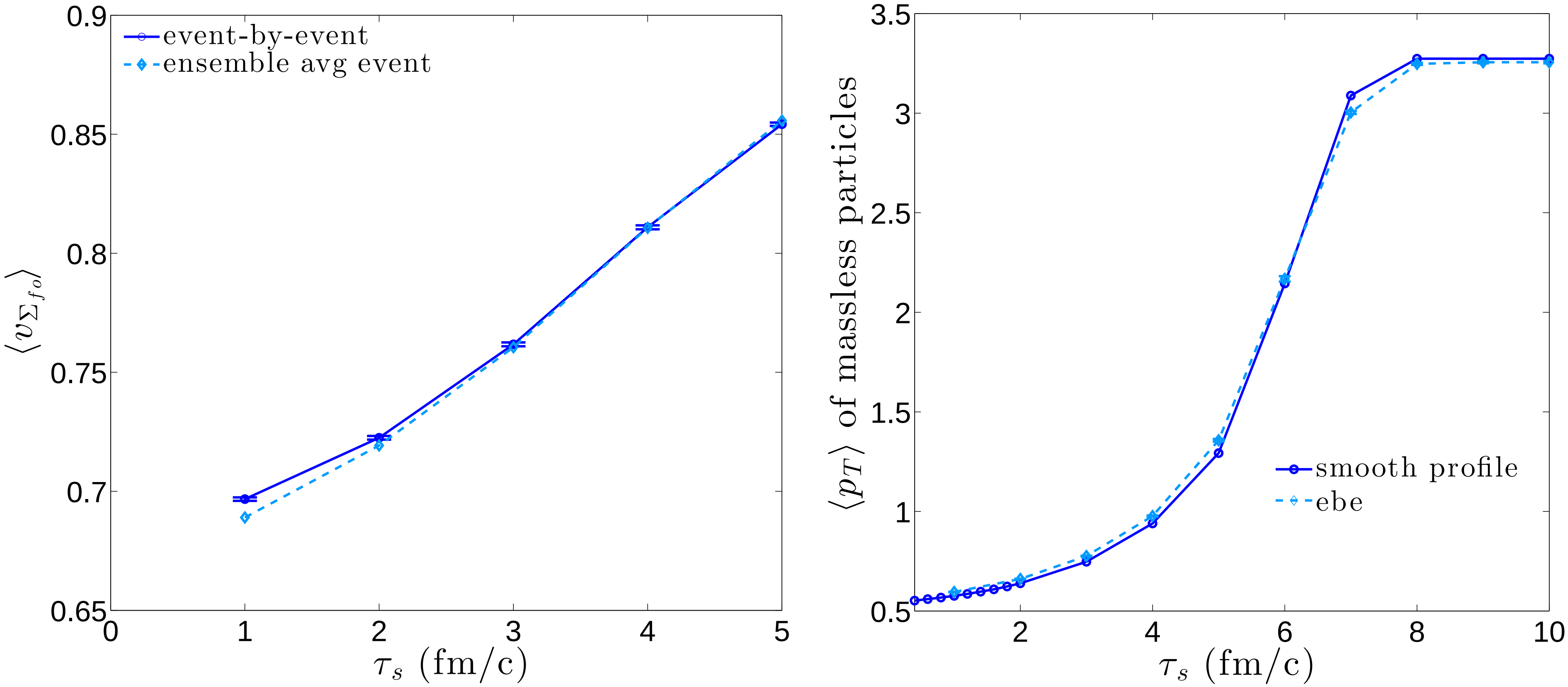
Pre-equilibrium dynamics (II)

Time evolution of radial flow for different switching times:



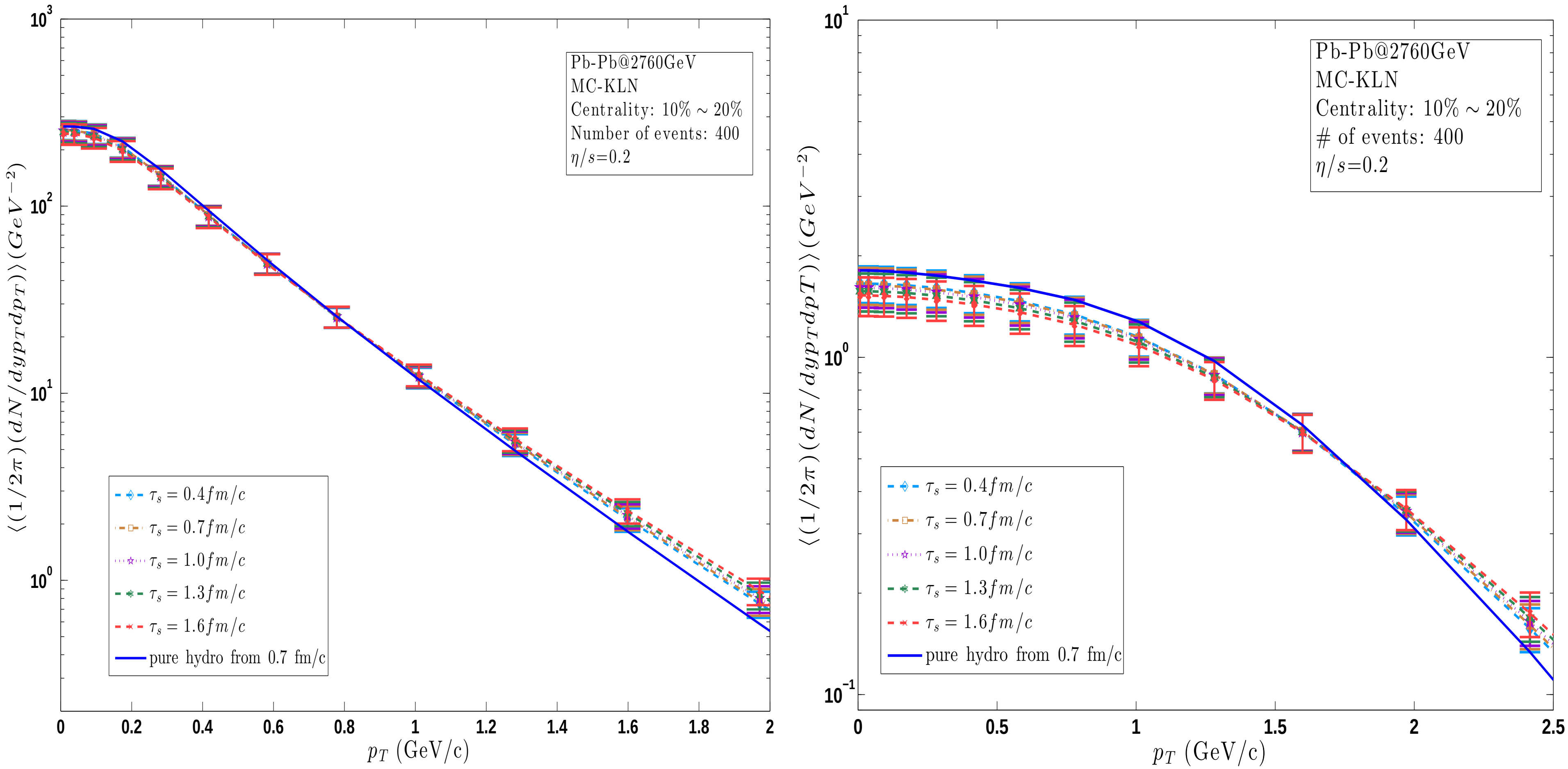
Pre-equilibrium dynamics (II)

Final radial flow and average p_T as function of switching time:



Pre-equilibrium dynamics (III)

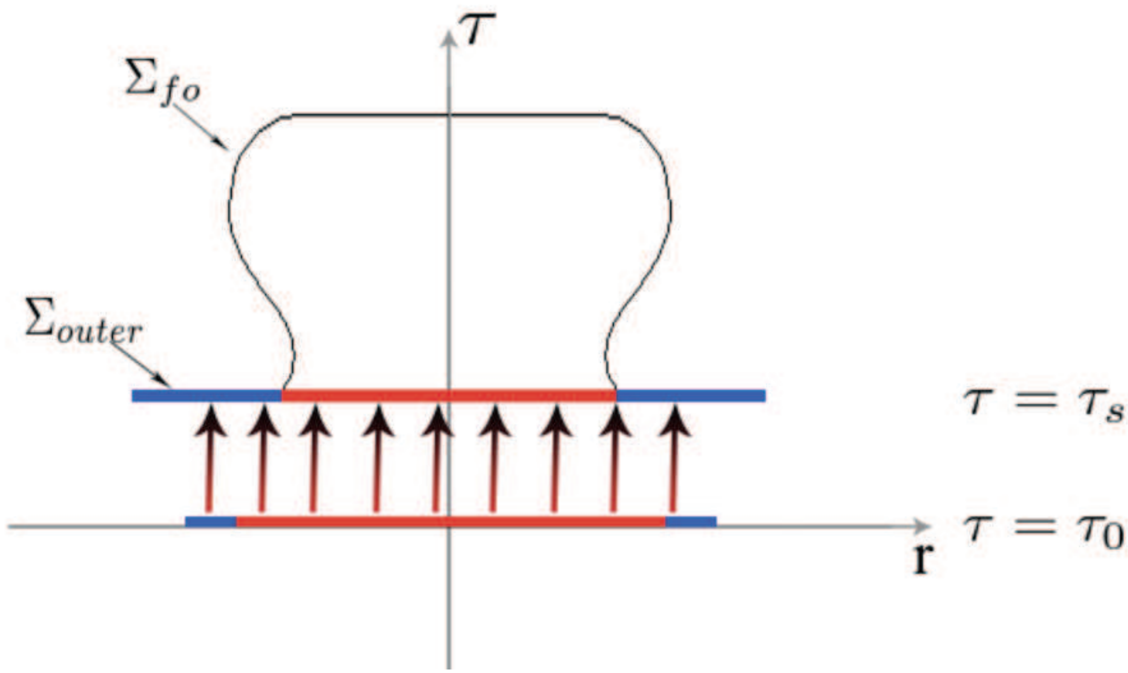
p_T-spectra for thermal pions (left) and thermal protons (right) (Jia Liu, 2013):



Late switching times $> 2\,\mathrm{fm}/c$ likely incompatible with experimental data.

The corona problem:

For late switching times, the contribution from corona particles that never thermalize can no longer be neglected:

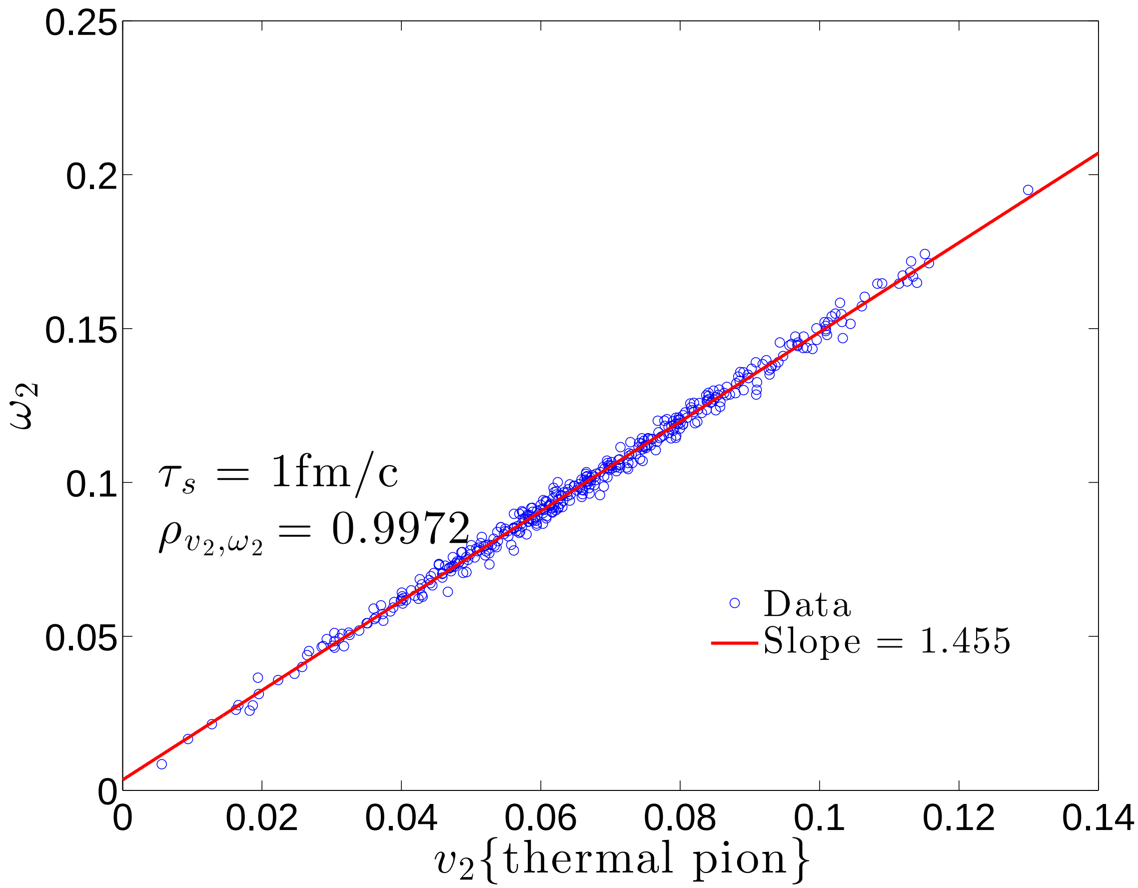


Problem: How to convert soft partons from the outer part of the hypersurface to hadrons?!

Way out: Use energy flow instead of particle flow to define anisotropic flow coefficients.

Pre-equilibrium dynamics (IV)

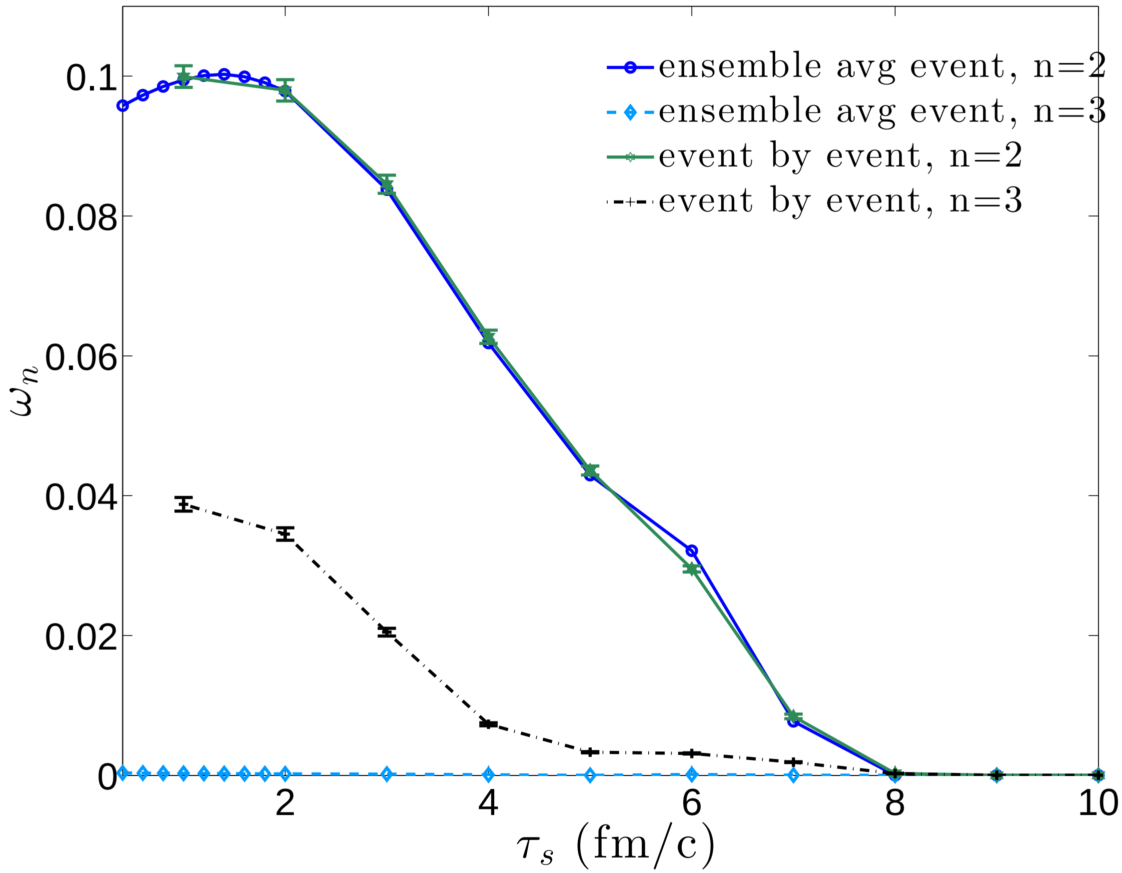
Energy anisotropic flow coefficients ω_2 as proxy for pion anisotropic flows v_2:



Similar correlation holds for ω_2 and proton v_2, and for triangular energy and particle flows.

Pre-equilibrium dynamics (V)

Final elliptic and triangular energy flow as function of switching time:



Less constraining than radial flow and p_T spectra.

Toy model for the source

$$\begin{aligned} S(x, K) &= \frac{S_0(K)}{\tau} \exp\left[-\frac{(\tau-\tau_f)^2}{2\Delta\tau^2} - \frac{(\eta-\eta_0)^2}{2\Delta\eta^2} \right. \\ &\quad -\frac{r^2}{2R^2}\left(1+2\bar{\epsilon}_3\cos(3(\phi-\bar{\psi}_3))\right) \\ &- \left. \frac{M_\perp}{T_0}\cosh(\eta-Y)\cosh\eta_t + \frac{K_\perp}{T_0}\cos(\phi-\Phi_K)\sinh\eta_t \right] \end{aligned}$$

where

$$\eta_t = \frac{\eta_f r}{R}\left(1+2\bar{v}_3\cos(3(\phi-\bar{\psi}_3))\right)$$

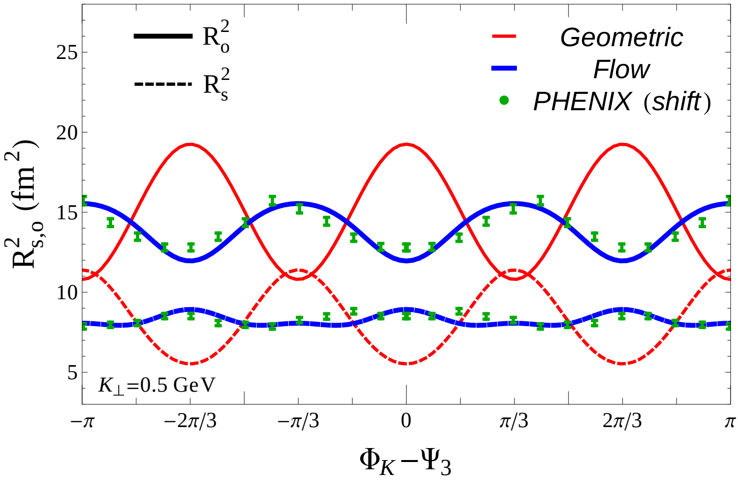
- $\bar{\epsilon}_3$: triangular azimuthal deformation
- $\bar{v}_3$: triangular flow deformation
- η_f: collective radial flow rapidity
- $\bar{\psi}_3$: triangular flow velocity angle, points in direction of largest flow rapidity and steepest descent of spatial density profile (note: $\Psi_n \neq \bar{\psi}_n$ in general)

HBT oscillation amplitudes: two examples

Hanbury Brown-Twiss (HBT) interferometry relative to the triangular flow plane in heavy-ion collisions

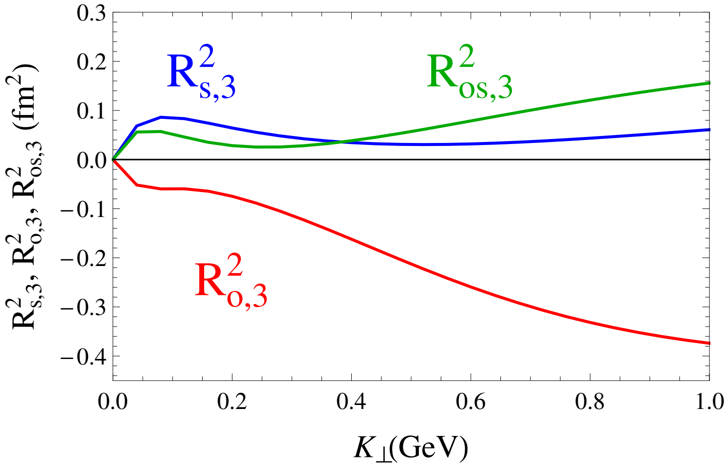
Christopher J. Plumberg *In collaboration with Chun Shen and Ulrich Heinz*

Phys. Rev. C 88, 044914 (2013)



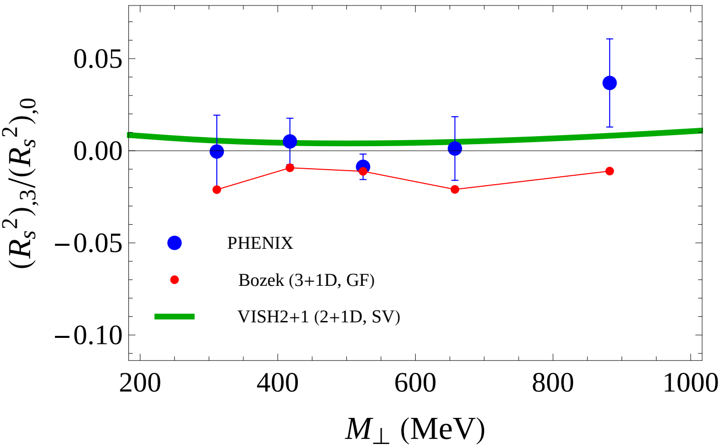
Hydrodynamic approach

$K_\perp$-dependence of $R^2_{ij,3}$ from hydrodynamics



Hydrodynamic approach

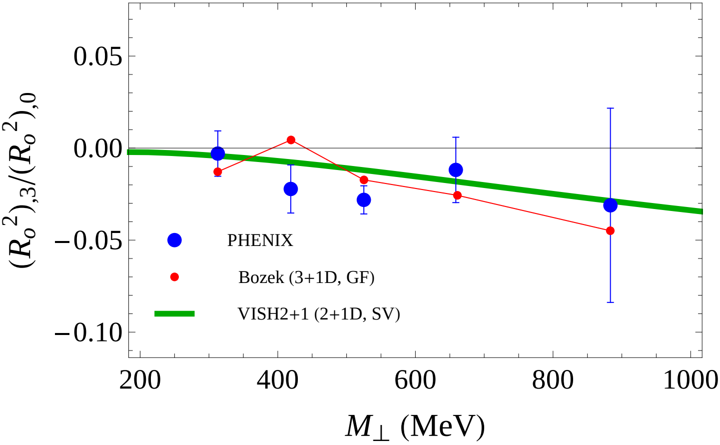
$M_\perp$-dependence of $R^2_{s,3}/R^2_{s,0}$ from hydrodynamics[1]



[1] arXiv:1401.7680, arXiv:1401.4894

Hydrodynamic approach

$M_\perp$-dependence of $R^2_{o,3}/R^2_{o,0}$ from hydrodynamics[2]



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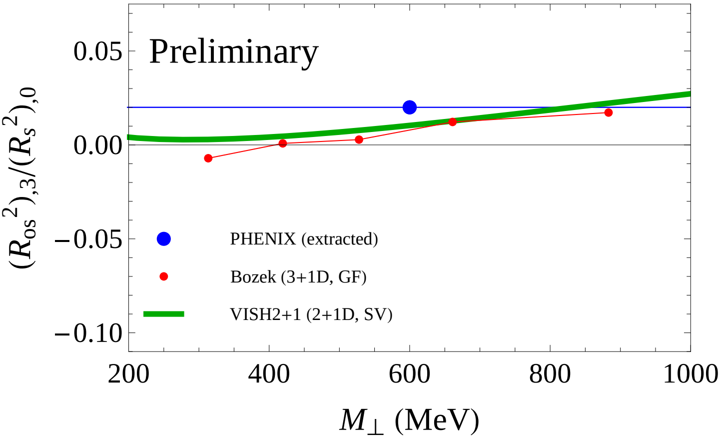
Hydrodynamic approach

$M_\perp$-dependence of $R^2_{os,3}/R^2_{s,0}$ from hydrodynamics[3]



[3] arXiv:1401.7680, arXiv:1401.4894

Part 2: Conclusions

- VISH2+1 qualitatively reproduces general trends of PHENIX data
- Qualitative features of $K_\perp$-dependence of hydrodynamic $R^2_{ij,3}$ similar to toy model for small $K_\perp$, more discrepancies at $K_\perp \gtrsim 0.3$ GeV
- Subtleties involving ensemble-averaging and the construction of the correlation function have not been addressed here

Anisotropic hydrodynamics (aHydro) (I)

Martinez and Strickland 2009

A non-perturbative method to account for large shear viscous effects stemming from large difference between longitudinal and transverse expansion rates.

$$f(x,p) = f_{\rm iso}\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde\mu(x)}{\Lambda(x)}\right) \equiv f_{\rm RS}(x,p)$$

where $\Xi^{\mu\nu}(x) = u^\mu(x)u^\nu(x) + \xi(x)z^\mu(x)z^\nu x$. (Romatschke&Strickland 2003)
3 flow and 3 "thermodynamic" parameters: $u^\mu(x)$; $\Lambda(x)$, $\tilde\mu(x)$, $\xi(x)$.

aHydro decomposition:

$$j^\mu_{\rm RS} = n_{\rm RS} u^\mu, \qquad T^{\mu\nu}_{\rm RS} = e_{\rm RS} u^\mu u^\nu - P_T \Delta^{\mu\nu} + (P_L - P_T) z^\mu z^\nu,$$

where, for massless partons ($m=0$), the effects of local momentum anisotropy can be factored out:

$$\begin{aligned} n_{\rm RS} &= \langle E \rangle_{\rm RS} = \mathcal{R}_0(\xi) n_{\rm iso}(\Lambda, \tilde\mu), \\ e_{\rm RS} &= \langle E^2 \rangle_{\rm RS} = \mathcal{R}(\xi) e_{\rm iso}(\Lambda, \tilde\mu), \\ P_{T,L} &= \langle p^2_{T,L} \rangle_{\rm RS} = \mathcal{R}_{T,L}(\xi) P_{\rm iso}(\Lambda, \tilde\mu). \end{aligned}$$

(See Strickland's talk for $\mathcal{R}$-functions.) The isotropic pressure is obtained from a locally isotropic EOS,

$$P_{\rm iso}(\Lambda, \tilde\mu) = P_{\rm iso}(e_{\rm iso}(\Lambda, \tilde\mu), n_{\rm iso}(\Lambda, \tilde\mu))$$

For massless noninteracting partons, $P_{\rm iso}(\Lambda, \tilde\mu) = \frac{1}{3} e_{\rm iso}(\Lambda, \tilde\mu)$ independent of chemical composition.

Viscous anisotropic hydrodynamics (vaHydro) (I)

$$f(x,p) = f_{\rm RS}(x,p) + \delta\tilde{f}(x,p) = f_{\rm iso}\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\Lambda(x)}\right) + \delta\tilde{f}(x,p)$$

Landau matching: $T^\mu_{\ \nu} u^\nu = e u^\mu$ with $u^\mu u_\mu = 1 \Longrightarrow$ fixes u^μ

no contribution to e, n from $\delta\tilde{f}$: $\langle E\rangle_{\tilde{\delta}} = \langle E\rangle_{\tilde{\delta}} = 0 \Longrightarrow$ fixes $\Lambda, \tilde{\mu}$.

no contribution to $P_T - P_L$ from $\delta\tilde{f}$: $\frac{x_\mu x_\nu + y_\mu y_\nu - 2 z_\mu z_\nu}{2} \langle p^{\langle\mu} p^{\nu\rangle}\rangle_{\tilde{\delta}} = 0 \Longrightarrow$ fixes ξ.

vaHydro decomposition:

$$j^\mu = j^\mu_{\rm RS} + \tilde{V}^\mu, \qquad \tilde{V}^\mu = \left\langle p^{\langle\mu\rangle}\right\rangle_{\tilde{\delta}},$$

$$T^{\mu\nu} = T^{\mu\nu}_{\rm RS} - \tilde{\Pi}\Delta^{\mu\nu} + \tilde{\pi}^{\mu\nu}, \qquad \tilde{\Pi} = -\frac{1}{3}\left\langle p^{\langle\alpha\rangle} p_{\langle\alpha\rangle}\right\rangle_{\tilde{\delta}}, \quad \tilde{\pi}^{\mu\nu} = \left\langle p^{\langle\mu} p^{\nu\rangle}\right\rangle_{\tilde{\delta}},$$

$u_\mu \tilde{\pi}^{\mu\nu} = \tilde{\pi}^{\mu\nu} u_\nu = (x_\mu x_\nu + y_\mu y_\nu - 2 z_\mu z_\nu)\tilde{\pi}^{\mu\nu} = \tilde{\pi}^\mu_\mu = 0 \Longrightarrow \tilde{\pi}^{\mu\nu}$ has 4 degrees of freedom.

Strategy: solve hydrodynamic equations for aHydro (which treat $P_T - P_L$ nonperturbatively) with added viscous flows from $\delta\tilde{f}$, together with IS-like "perturbative" equations of motion for $\tilde{\Pi}$, $\tilde{V}^\mu$, $\tilde{\pi}^{\mu\nu}$.

Viscous anisotropic hydrodynamics (vaHydro) (II)

Hydrodynamic equations of motion:

$\partial_\mu j^\mu = C \equiv \int_p C(x,p) \Longrightarrow \dot{n}_{\rm RS} = -n_{\rm RS}\theta - \partial_\mu \tilde{V}^\mu + \frac{n_{\rm RS}-n_{\rm iso}}{\tau_{\rm rel}}$ in RTA

$\partial_\mu T^{\mu\nu} = 0 \Longrightarrow$

$$\dot{e} = -(e+P_T)\theta_\perp - (e+P_L)\frac{u_0}{\tau} - \tilde{\Pi}\theta + \tilde{\pi}^{\mu\nu}\sigma_{\mu\nu},$$

$$(e+P_T+\tilde{\Pi})\dot{u}_\perp = -\partial_\perp(P_T+\tilde{\Pi}) - u_\perp(\dot{P}_T+\dot{\tilde{\Pi}}) - u_\perp(P_T-P_L)\frac{u_0}{\tau} + \left(\frac{u_x\Delta^1{}_\nu + u_y\Delta^2{}_\nu}{u_\perp}\right)\partial_\mu\tilde{\pi}^{\mu\nu},$$

$$(e+P_T+\tilde{\Pi})u_\perp\dot{\phi}_u = -D_\perp(P_T+\tilde{\Pi}) - \frac{u_y\partial_\mu\tilde{\pi}^{\mu 1} - u_x\partial_\mu\tilde{\pi}^{\mu 2}}{u_\perp},$$

where $\theta_\perp = \partial_\tau u_0 + \nabla_\perp\cdot\boldsymbol{u}_\perp$ and $D_\perp = (u_x\partial_y - u_y\partial_x)/u_\perp$.

To derive **equations of motion for** $\tilde{\Pi}$, $\tilde{\boldsymbol{V}}^\mu$, **and** $\tilde{\pi}^{\mu\nu}$, we follow DMNR (2012). Ignoring heat conduction by setting $\tilde{\mu} = 0$ and taking $m = 0$ we find (Bazow, UH, Strickland, 1311.6720)

$$\dot{\tilde{\pi}}^{\mu\nu} = -2\dot{u}_\alpha\tilde{\pi}^{\alpha(\mu}u^{\nu)} - \frac{1}{\tau_{\rm rel}}\Big[(P-P_T)\Delta^{\mu\nu} + (P_L-P_T)z^\mu z^\nu + \tilde{\pi}^{\mu\nu}\Big] + \mathcal{K}_0^{\mu\nu} + \mathcal{L}_0^{\mu\nu} + \mathcal{H}_0^{\mu\nu\lambda}\dot{z}_\lambda$$
$$+\mathcal{Q}_0^{\mu\nu\lambda\alpha}\nabla_\lambda u_\alpha + \mathcal{X}_0^{\mu\nu\lambda}u^\alpha\nabla_\lambda z_\alpha - 2\lambda^0_{\pi\pi}\tilde{\pi}^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda + 2\tilde{\pi}^{\lambda\langle\mu}\omega^{\nu\rangle}_\lambda - 2\delta^0_{\pi\pi}\tilde{\pi}^{\mu\nu}\theta,$$

Test of vaHydro: (0+1)-dimensional expansion (I)

As you heard in Mike Strickland's talk, for (0+1)-d (longitudinally boost-invariant) expansion, the BE can be solved exactly in RTA, and the solution can be used to test the various macroscopic hydrodynamic approximation schemes.

Setting homogeneous initial conditions in r and η_s and zero transverse flow, $\tilde{\pi}^{\mu\nu}$ reduces to a single non-vanishing component $\tilde{\pi}$: $\tilde{\pi}^{\mu\nu} = \mathrm{diag}(0, -\tilde{\pi}/2, -\tilde{\pi}/2, \tilde{\pi})$ at $z = 0$.

We use the factorization $n_{\mathrm{RS}}(\xi\Lambda) = \mathcal{R}_0(\xi) n_{\mathrm{iso}}(\Lambda)$ etc. to get EOMs for $\dot{\xi}$, $\dot{\Lambda}$, $\dot{\tilde{\pi}}$:

$$\frac{\dot{\xi}}{1{+}\xi} - 6\frac{\dot{\Lambda}}{\Lambda} = \frac{2}{\tau} + \frac{2}{\tau_{\mathrm{rel}}}\left(1 - \sqrt{1{+}\xi}\,\mathcal{R}^{3/4}(\xi)\right) ,$$

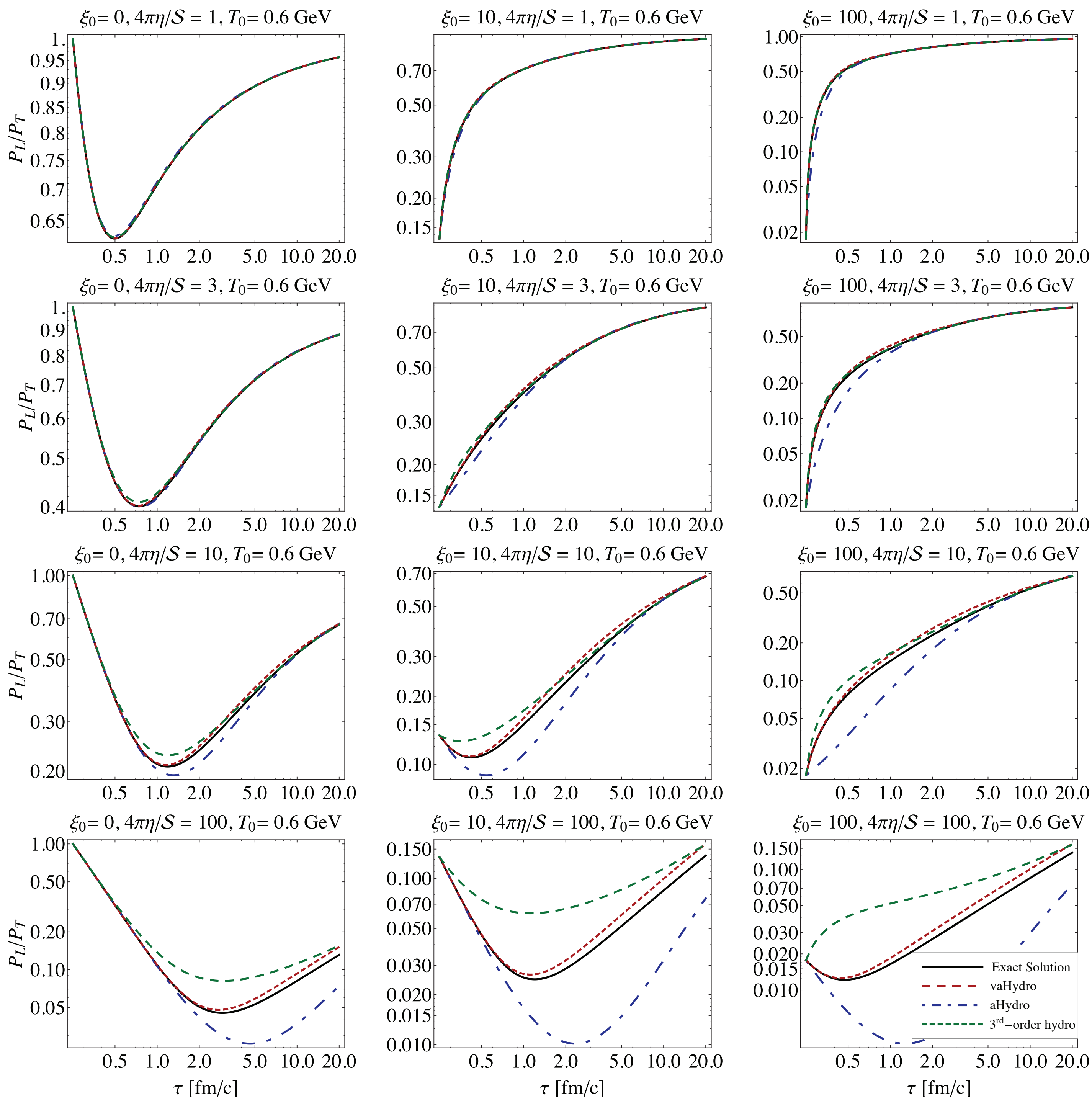
$$\mathcal{R}'(\xi)\dot{\xi} + 4\mathcal{R}(\xi)\frac{\dot{\Lambda}}{\Lambda} = -\left(\mathcal{R}(\xi) + \frac{1}{3}\mathcal{R}_L(\xi)\right)\frac{1}{\tau} + \frac{\tilde{\pi}}{e_{\mathrm{iso}}(\Lambda)\tau},$$

$$\dot{\tilde{\pi}} = -\frac{1}{\tau_{\mathrm{rel}}}\left[\left(\mathcal{R}(\xi) - \mathcal{R}_{\mathrm{L}}(\xi)\right)P_{\mathrm{iso}}(\Lambda) + \tilde{\pi}\right] - \frac{38}{21}\frac{\tilde{\pi}}{\tau}$$
$$+12\left[\frac{\dot{\Lambda}}{\Lambda}\left(\mathcal{R}_{\mathrm{L}}(\xi) - \frac{1}{3}\mathcal{R}(\xi)\right) + \left(\frac{1{+}\xi}{\tau} - \frac{\dot{\xi}}{2}\right)\left(\mathcal{R}_{-1}^{zzzz}(\xi) - \frac{1}{3}\mathcal{R}_{1}^{zz}(\xi)\right)\right]P_{\mathrm{iso}}(\Lambda),$$

τ_{del} and η/s are related by (Denicol, Koide, Rischke, PRL 105 (2010))

$$\tau_{\mathrm{rel}} = 5\frac{\eta/s}{T} = 5\frac{\eta/s}{\mathcal{R}^{1/4}(\xi)\Lambda}$$

We solve these equations and compare with the exact solution:



$\xi_0 = 0, 4\pi\eta/\mathcal{S} = 1, T_0 = 0.6$ GeV
$\xi_0 = 10, 4\pi\eta/\mathcal{S} = 1, T_0 = 0.6$ GeV
$\xi_0 = 100, 4\pi\eta/\mathcal{S} = 1, T_0 = 0.6$ GeV
$\xi_0 = 0, 4\pi\eta/\mathcal{S} = 3, T_0 = 0.6$ GeV
$\xi_0 = 10, 4\pi\eta/\mathcal{S} = 3, T_0 = 0.6$ GeV
$\xi_0 = 100, 4\pi\eta/\mathcal{S} = 3, T_0 = 0.6$ GeV
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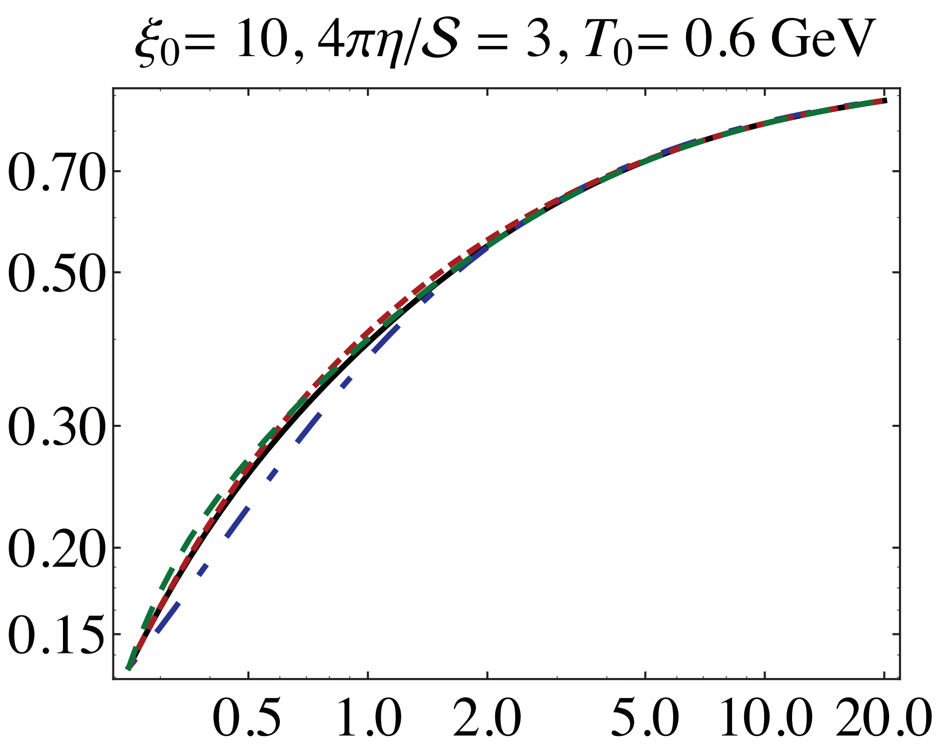
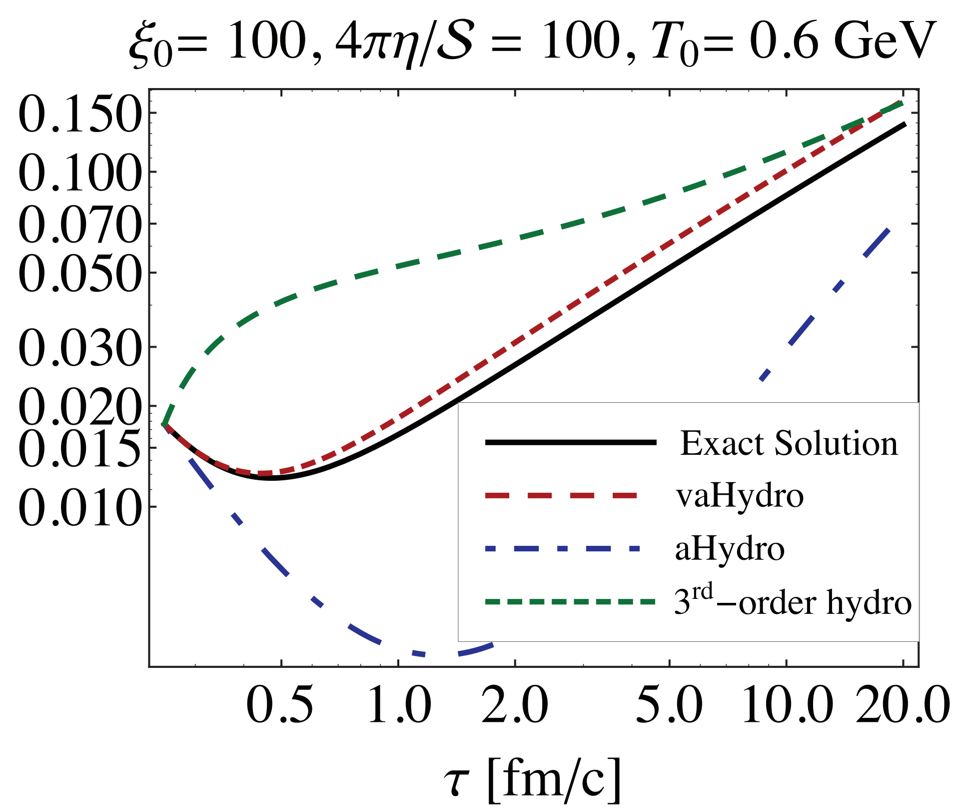
$\xi_0 = 100, 4\pi\eta/\mathcal{S} = 100, T_0 = 0.6$ GeV
P_L/P_T
τ [fm/c]
Exact Solution
vaHydro
aHydro
3rd–order hydro

Test of vaHydro: (0+1)-dimensional expansion (II)

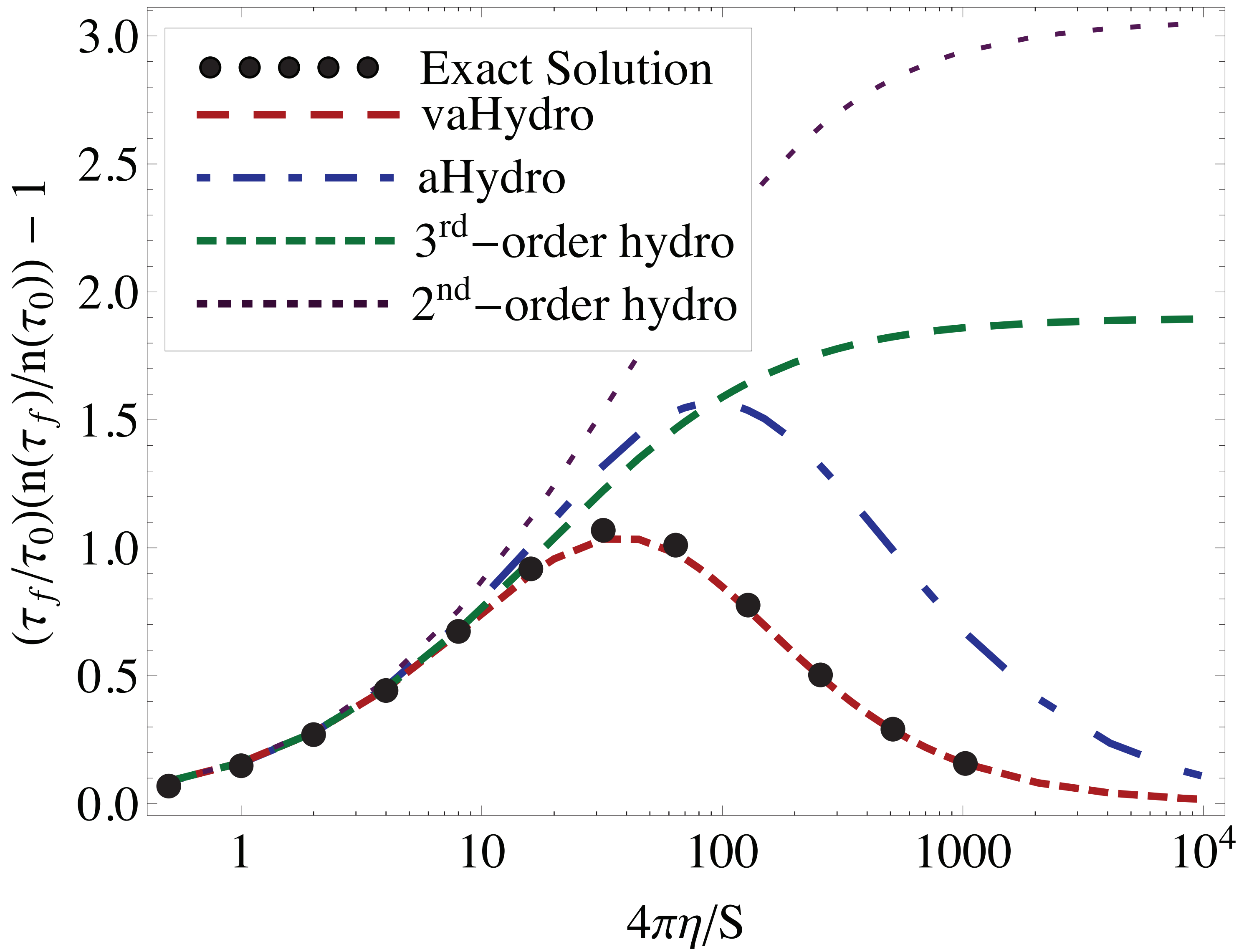
Pressure anisotropy P_L/P_T vs. τ:





Test of vaHydro: (0+1)-dimensional expansion (III)

Total entropy (particle) production $\frac{n(\tau_f)\cdot\tau_f}{n(\tau_0)\cdot\tau_0} - 1$



Advantages of vaHydro

- For early times and/or near the transverse edge in heavy-ion collision fireballs, rapid longitudinal expansion generates large inverse Reynolds numbers for the shear pressure, $\mathrm{R}_\pi^{-1} = \sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}/P_{\mathrm{iso}}$, causing Israel-Stewart second order viscous hydrodynamics to break down.
- The large local pressure anisotropies caused by a large difference in longitudinal and transverse expansion rates can be treated efficiently by using the non-perturbative aHydro approach which is based on an expanseion around a locally spheroidally deformed distribution f_{RS}.
- This strongly reduces the shear inverse Reynolds numbers $\tilde{\mathrm{R}}_\pi^{-1} = \sqrt{\tilde{\pi}^{\mu\nu}\tilde{\pi}_{\mu\nu}}/\mathcal{P}_{\mathrm{iso}}$ associated with the remaining shear stress tensor $\tilde{\pi}^{\mu\nu}$ resulting from the much smaller deviation $\delta\tilde{f}$ of the local distribution function from f_{RS}.
- vaHydro combines the advantages of aHydro with a complete (although perturbative) second-order treatment of all remaining viscous effects à la Israel-Stewart.
- In a test of (0+1)-d expansion, which maximizes the difference between longitudinal and transverse expansion rates, against an exact solution of the Boltzmann equation, vaHydro outperforms all other known hydrodynamic approximation schemes by a considerable margin.
- This should open the door in (3+1)-d systems to match microscopic pre-equilibrium theories to viscous hydrodynamics at earlier times than possible with IS-theory and its variants.
- By replacing $f = f_{\mathrm{eq}} + \delta f$ by $f = f_{\mathrm{RS}} + \delta\tilde{f}$ we should be able to reduce uncertainties related to δf corrections to the momentum distributions at freeze-out (or, for photons, everywhere)

To do list:

The to-do list for the next year – Pt I

- After some discussions with the bulk WG members, I have come up with a to-do list for the next year.
- I have also taken the liberty to add some things that I find particularly interesting.
- The list I present is by no means a prioritized list.

- Complete event-by-event all-stage dynamical simulations with fluctuating initial conditions
- Completion of the jet quenching module (jet shower MC) and couple it with iEBE (mostly work needed by the jet WG)
- Completion and publication of the iEBE documentation and the code package (mostly done already)
- 2+1d and 3+1d NLO aHydro with fluctuating ICs (aka vaHydro)
- Lots of uncertainties associated with freeze-out. This is important for how we fix the physical parameters that are used at all times during the bulk evolution. Needs some critical attention.

The to-do list for the next year – Pt II

- Anisotropic freezeout; instead of using linearly-corrected distribution functions, use anisotropically deformed distribution functions
- Systematic studies of pre-equilibrium dynamics on final observables
- Implementation and testing of the self-consistent initial conditions (flow & rapidity dependence) from the CGC
- More studies of the impact of viscous (anisotropic) corrections to electromagnetic signatures → necessary for experimental determination of the degree of isotropization of the QGP
- Work needed on elimination of instabilities in the relativistic Lattice Boltzmann solvers; work in progress at Colorado to implement “f0 stabilization”
- Squeeze B. Schenke hard to provide 2+1d and 3+1d MUSCL-based hydro as an alternative to the current VISHNU hydro module. Important to test dependence of results on the underlying hydro module.

Supplements

4. Influence of pre-equilibrium stage (3)

- Construct anisotropy from E_T distribution
  - Good news: free-streamed distribution is known

$$\frac{dE_T}{dyd\phi} = \sum_i \frac{g_i}{(2\pi)^3} \int p^0 p_\perp dp_\perp \int_\Sigma p^\mu d^3\sigma_\mu f_i(x,p) \ \ \text{(i for parton or hadron species)}$$

  - Apply to freeze-out surface:

$$\left.\frac{dE_T}{dyd\phi}\right|_{\Sigma_{fo}} = \sum_i \frac{g_i}{(2\pi)^3} \int p^0 p_\perp dp_\perp \int_{\Sigma_{fo}} p^\mu d^3\sigma_\mu (f_{i,eq} + \delta f_i)$$

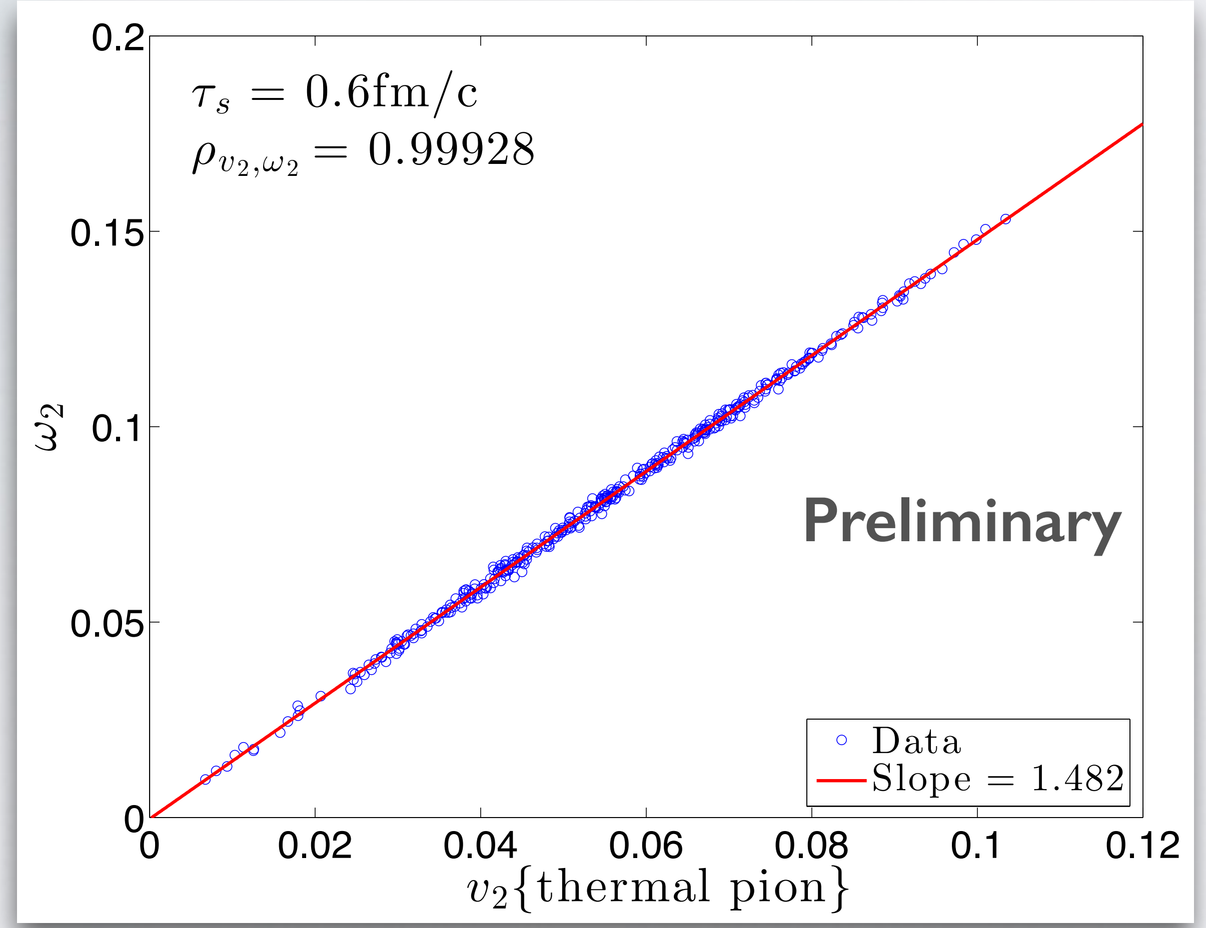
$$\boxed{\omega_n e^{in\tilde{\Psi}_n} = \frac{\int_\Sigma \frac{dE_T}{dyd\phi} e^{in\phi} d\phi}{\int_\Sigma \frac{dE_T}{dyd\phi} d\phi}}$$

  - Apply to outer surface:

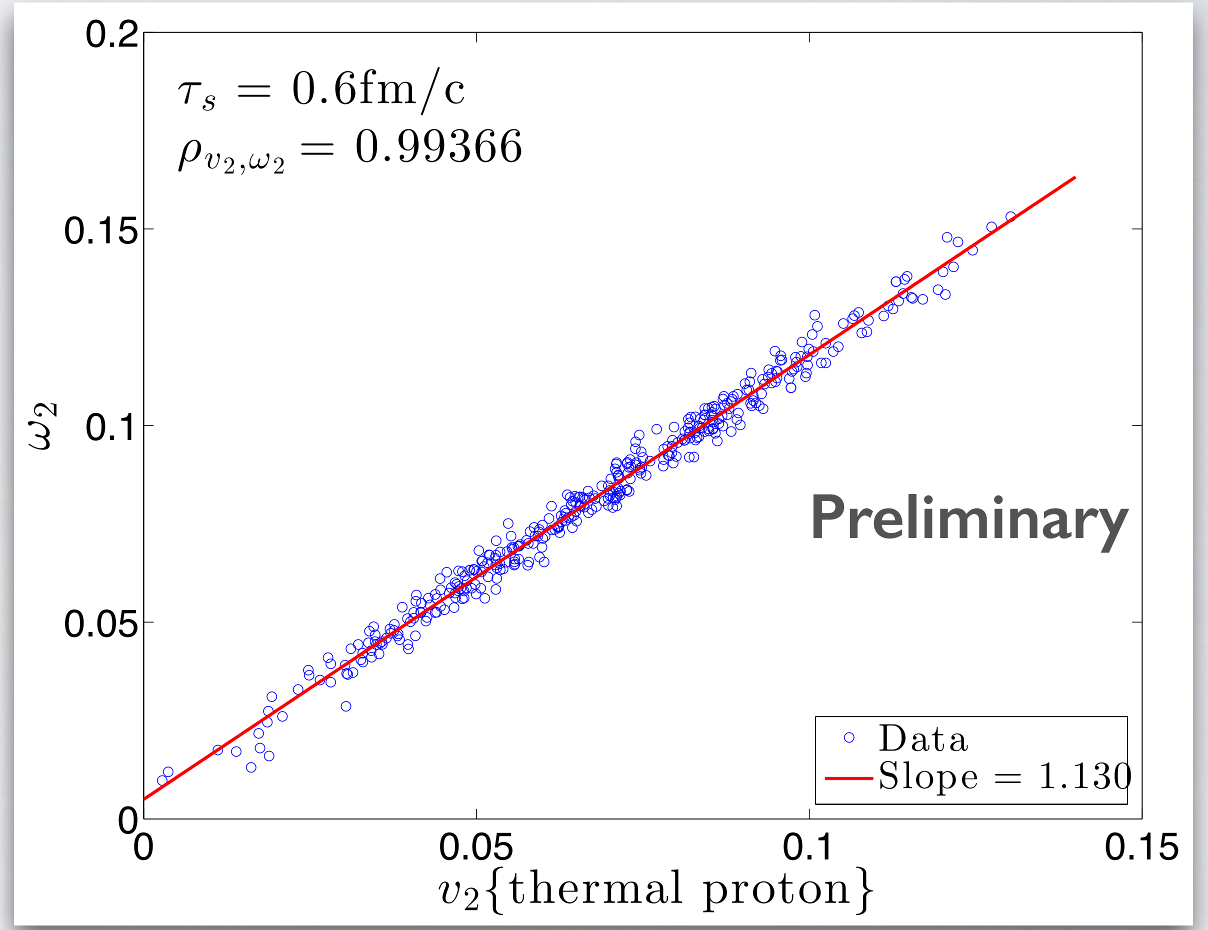
$$\left.\frac{dE_T}{dyd\phi}\right|_{\Sigma_{outer}} = \sum_i \frac{g_i}{(2\pi)^3} \int d^2x_\perp \int p_\perp^2 dp_\perp f_i(x,p)$$

4. Influence of pre-equilibrium stage (4)

- Correlation with flow anisotropy v_2



Pion +

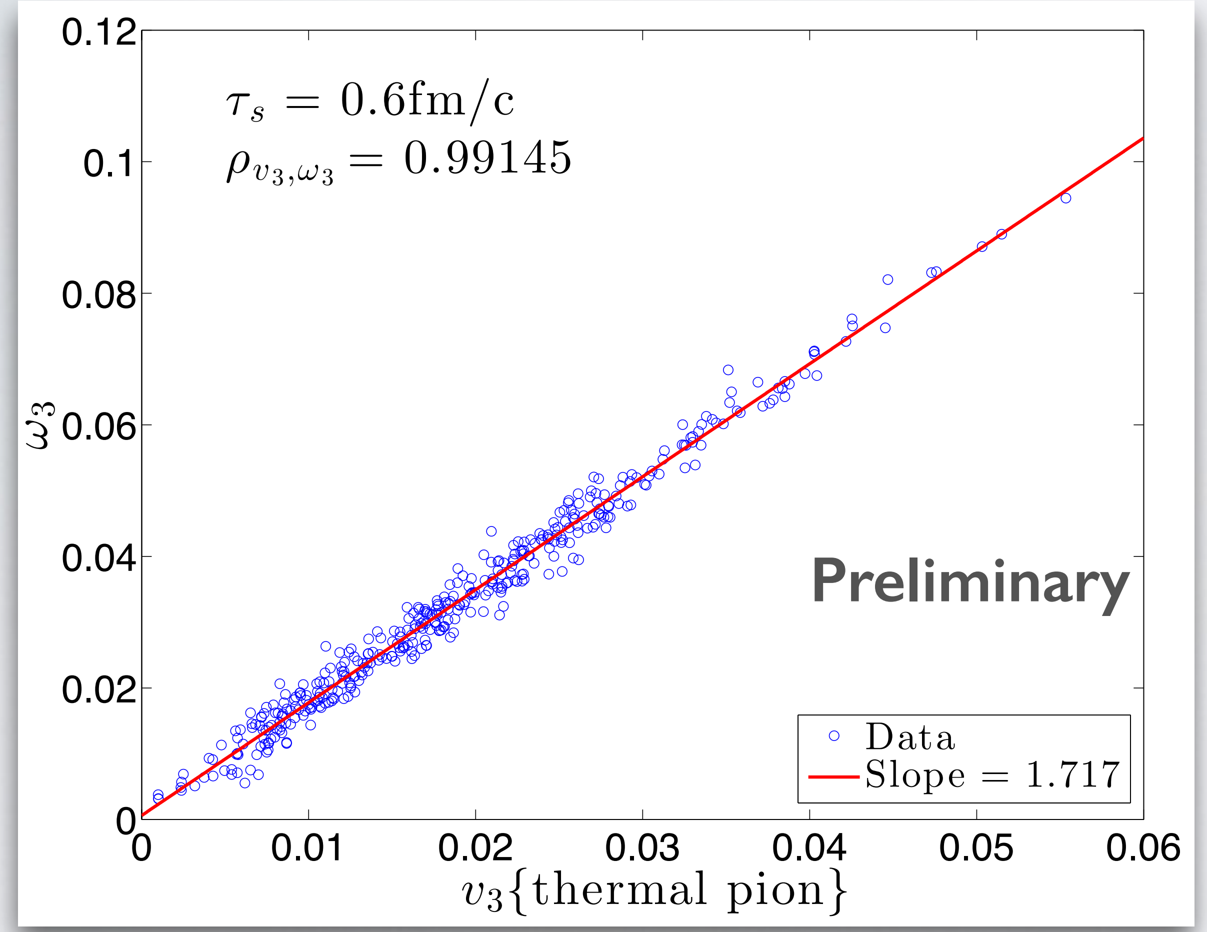


Proton

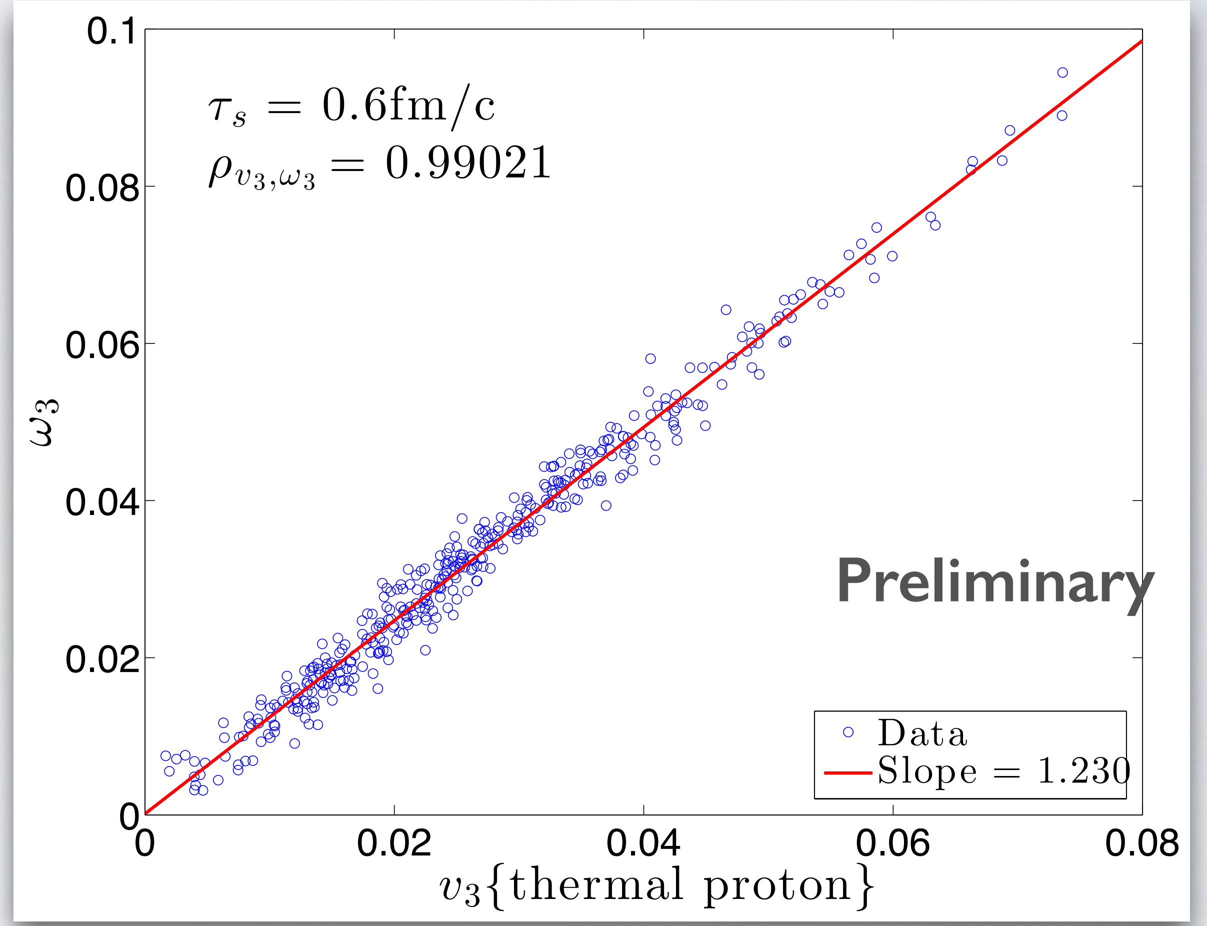
- Early matching time: not so much cells move out
- Strong correlation!

4. Influence of pre-equilibrium stage (4)

- Correlation with flow anisotropy v_3



Pion +



Proton

- Correlation is still good.