Introduction to pQCD and Jets: lecture 3

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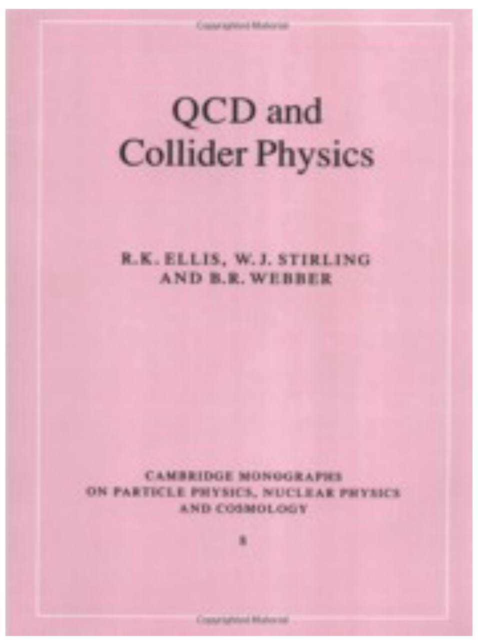
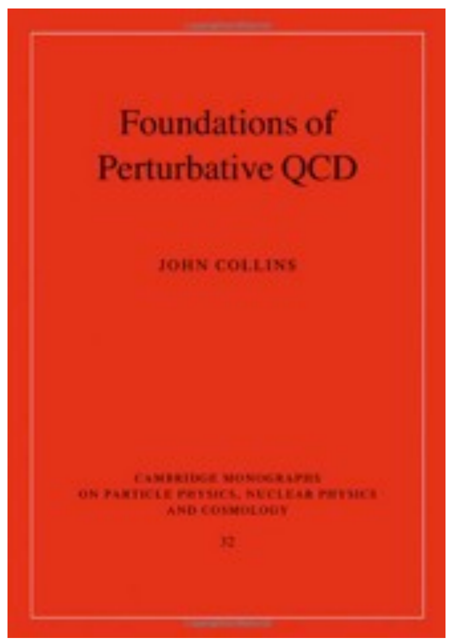
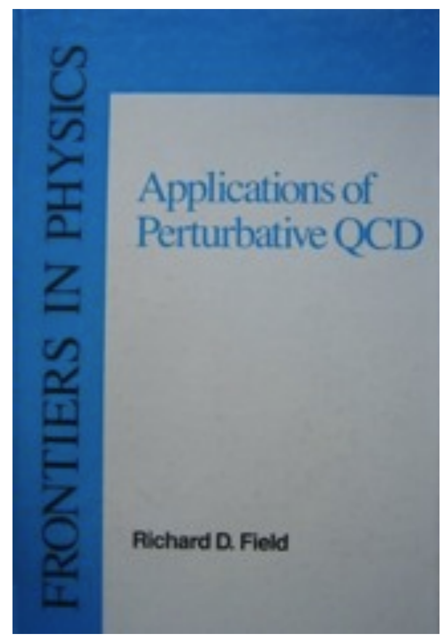
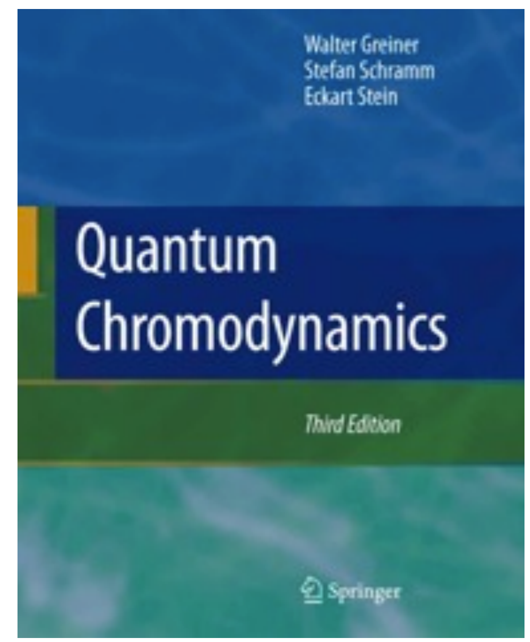
Selected references on QCD

- QCD and Collider Physics: Ellis-Stirling-Webber
- Foundations of Perturbative QCD: J. Collins
- Applications of Perturbative QCD: R. Field
- Quantum Chromodynamics: Greiner-Schramm-Stein





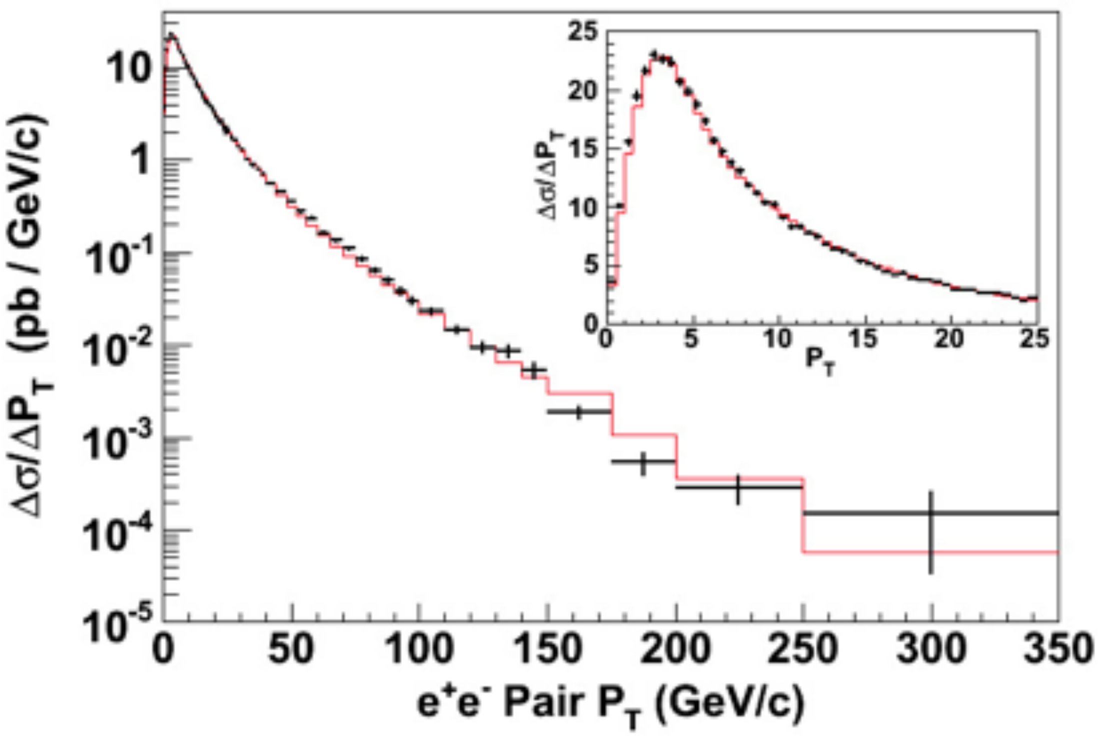




- CTEQ collaboration: http://www.phys.psu.edu/~cteq
- QCD Resource Letter: arXiv:1002.5032 by Kronfeld-Quigg
- Particle Data Group: http://pdg.lbl.gov

Recap - lecture 2: QCD foundation and multiple scattering

- pQCD factorization: collinear vs TMD factorization



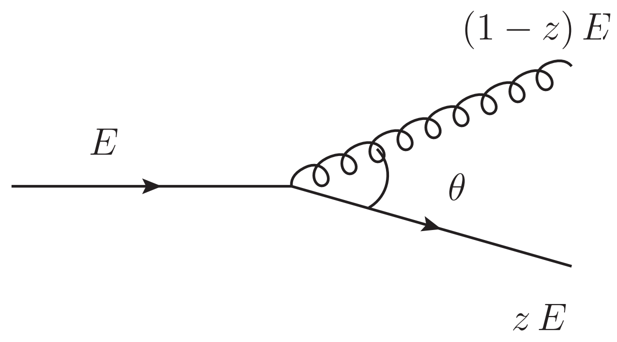
- Parton multiple scattering is very important in understanding nontrivial nuclear dependence
  - Can be described in a high-twist expansion formalism when "kt" is reasonably small
  - Have to recover the full "kt" dependence (resummed to Wilson line) when "kt" is the main degree of freedom (e.g., in small-x region)
- Factorization at twist-4 has been verified up to one-loop order

Why Jets?

- In QCD Lagrangian quarks and gluons are the degrees of freedom, so pQCD calculation deals with quarks and gluons only. However, quarks and gluons are never observed into detectors
- QCD final states involve highly collimated sprays of energetic hadrons, a.k.a. jets
- Jets are the footprints of partons in the detector

Footprints of partons

- Accelerated quarks always radiate gluons

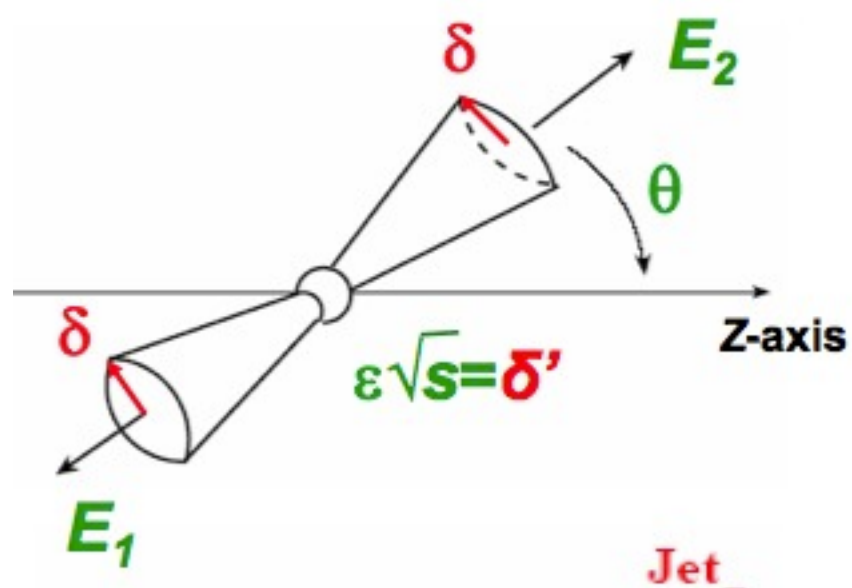


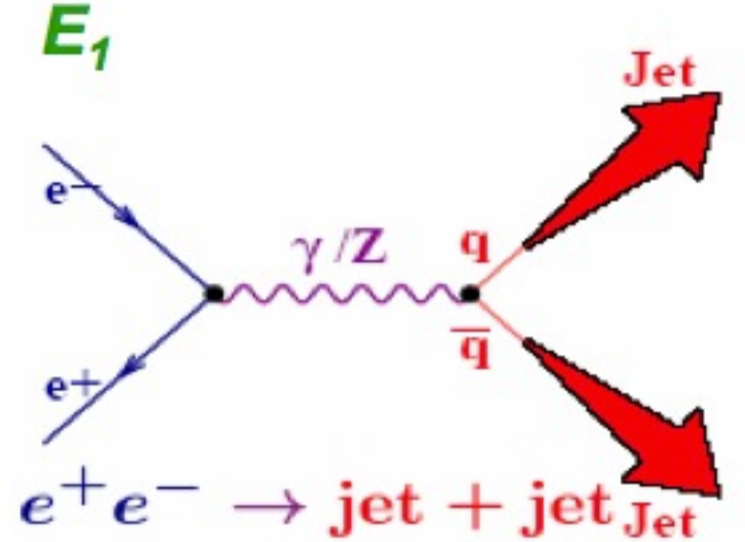
$$dP_{q\to qg} = \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2} dz C_F \frac{1+z^2}{1-z}$$

- The structure of the jet reflects the properties of underlying QCD radiation
  - High probability of emission of soft (z→1) and collinear (θ→0) gluons
  - Extra hard gluon emission ~ αs(E) (strong coupling constant)
- Asymptotic freedom: αs(E)→0 for E→∞, thus the higher the energy the more collimated the jets
- Jet cross sections are computable in perturbative QCD using the degrees of freedom of quarks and gluons. Even though experimentally reconstructed jets through the hadrons, experiments and theory should be the same

How to define a jet: original one

- Sterman-Weinberg jets
  - Only defined in e+e- process
  - Cones of opening angle δ containing all but a fraction ε of the total energy in the event





$$\sigma_{2\mathrm{Jet}}^{(\mathrm{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta')+3\ln(\delta)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$

Modern jet algorithms

- Cone algorithms
- Successive recombination algorithms

Cone algorithms

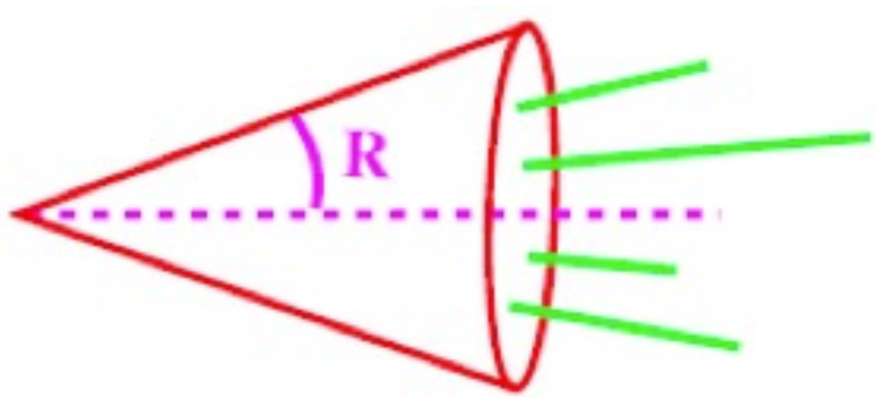
- Particles in cone of size R, defined in angular space (η,ϕ)
  - The jet is defined by the particles inside a circle in the plane formed by rapidity and azimuthal angle, such that the sum of the four momenta of these particles points in the direction of its center
  - Particle j is inside the cone iff

$$R_{jC}^2 = (\eta_j - \eta_C)^2 + (\phi_j - \phi_C)^2 \leq R^2$$

  - The jet axis

$$\eta_J = \frac{\sum_i E_T^i \eta_i}{\sum_i E_T^i} \qquad \phi_J = \frac{\sum_i E_T^i \phi_i}{\sum_i E_T^i}$$

  - Jet is defined a "stable" cone if jet axis is coincident with the cone centroid



Successive recombination algorithms: kt-type

- Define a distance measure between particles j and k, also w.r.t. the beam

$$d_{ij} \equiv \min(k_{T_i}^{2p}, k_{T_j}^{2p}) \frac{R_{ij}^2}{R^2} \qquad R_{ij}^2 \equiv (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} \equiv k_{T_i}^{2p}$$

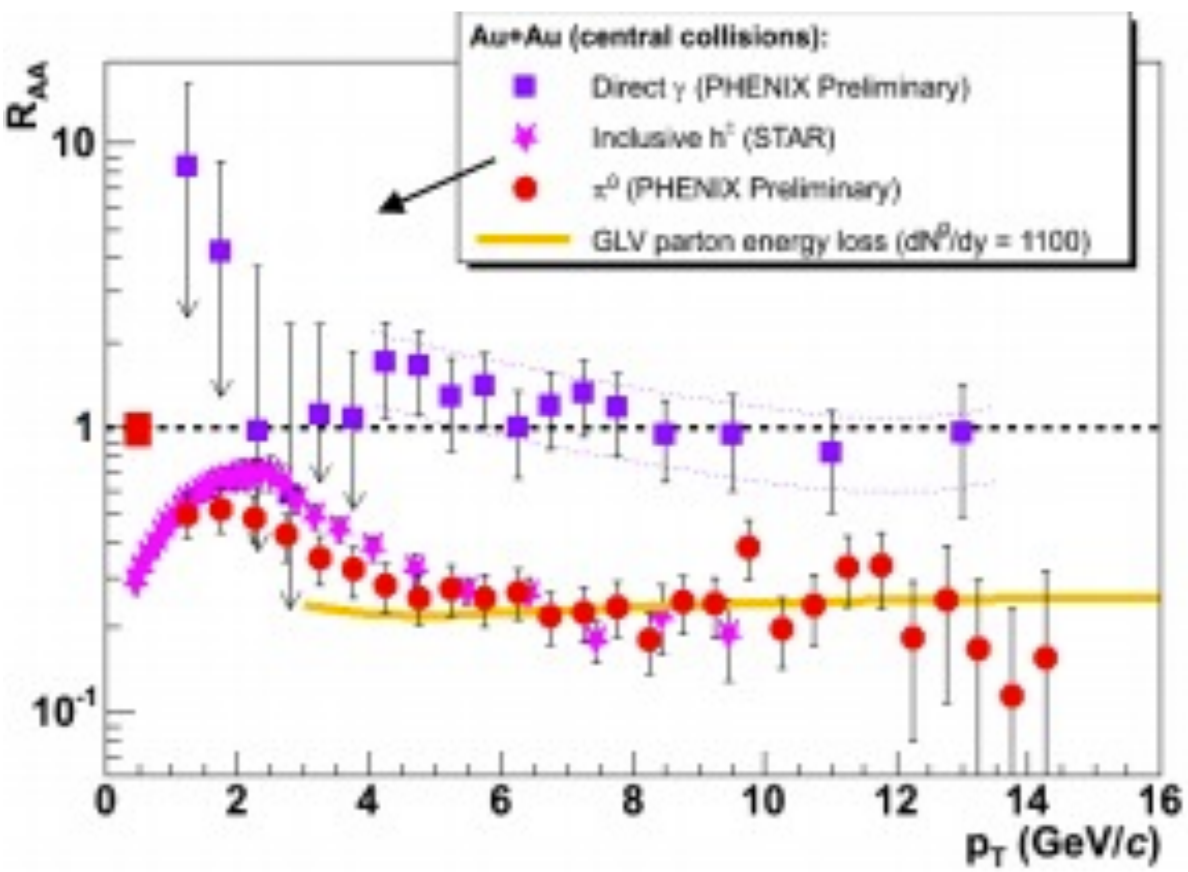
Find the minimum $d_{\min}$ of all the d_{ij}, d_{iB}. If $d_{\min}$ is a d_{ij} merge particles i and j into a single particle, summing their four-momenta (this is E-scheme recombination); if it is a d_{iB} then declare particle i to be a final jet and remove it from the list.

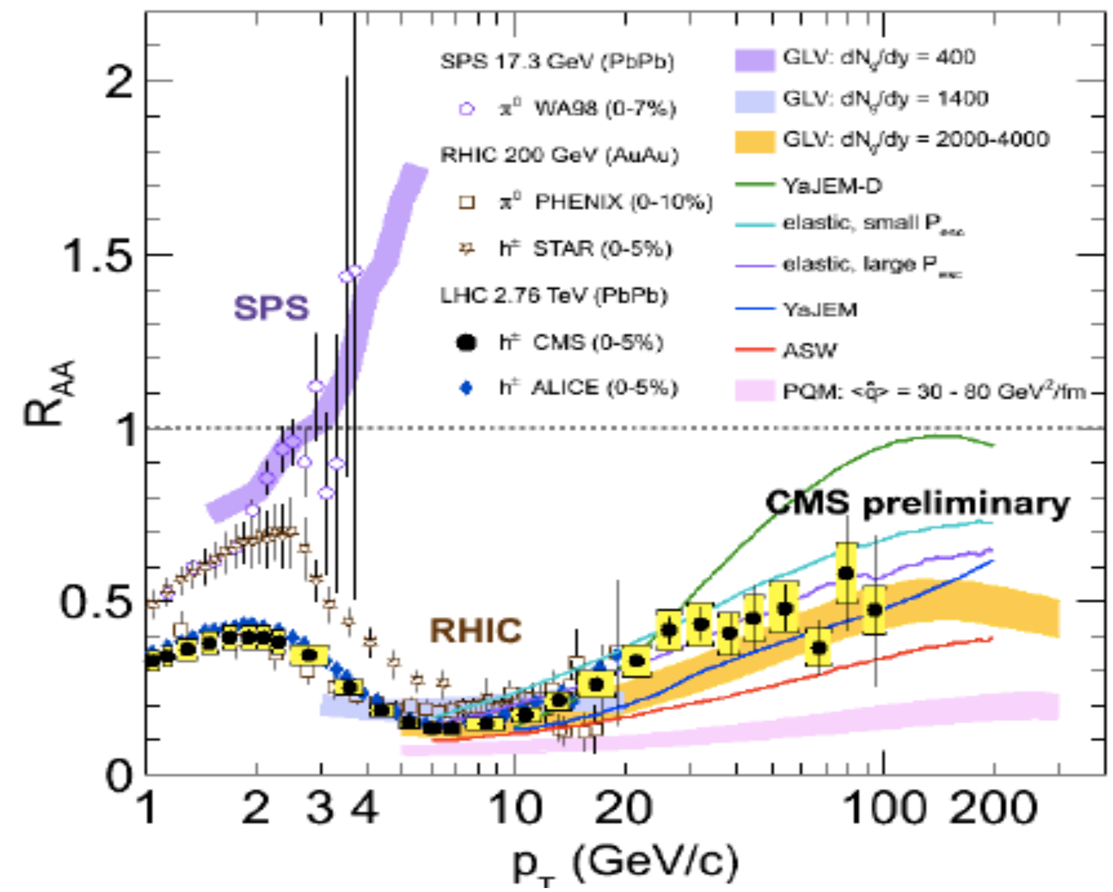
- Repeat above procedure until no particles are left
- p=1, kt-algorithm; p=-1 anti-kt algorithm

Leading light hadron suppression

- Jet quenching for light hadron production in both RHIC and LHC

$$R_{AA} = \frac{\mathrm{Yield}_{AuAu} / \langle N_{binary} \rangle_{AuAu}}{\mathrm{Yield}_{pp}}$$





- Light hadron comes from the fragmentation of light (massless) quarks and gluons

What about heavy quark?

- Quark mass effect in radiated gluon radiation: the so-called dead-cone effect which leads to less radiative energy loss for heavy quarks

$$dP_{\rm HQ} = dP_0 \cdot \left(1 + \frac{\theta_0^2}{\theta^2}\right)^{-2} \qquad \theta_0 \equiv \frac{M}{E}$$

"Heavy Quark Calorimetry of QCD Matter",
Dokshitzer and Kharzeev
hep-ph/0106202 (2001)

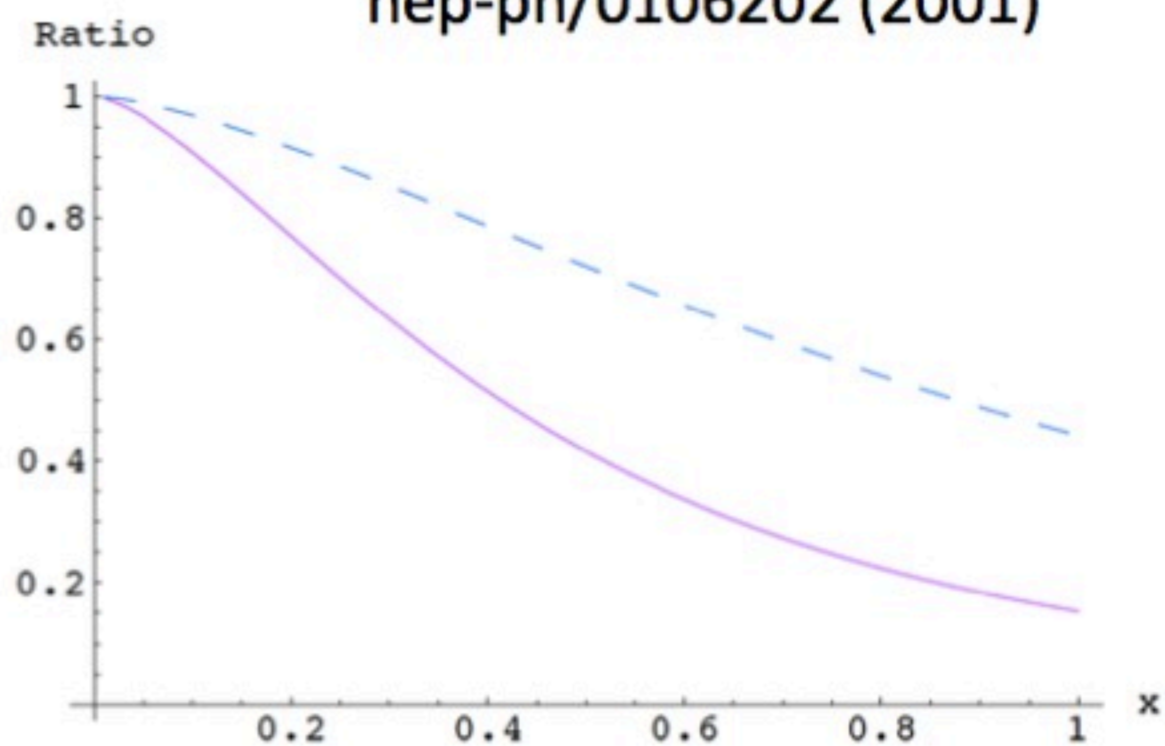
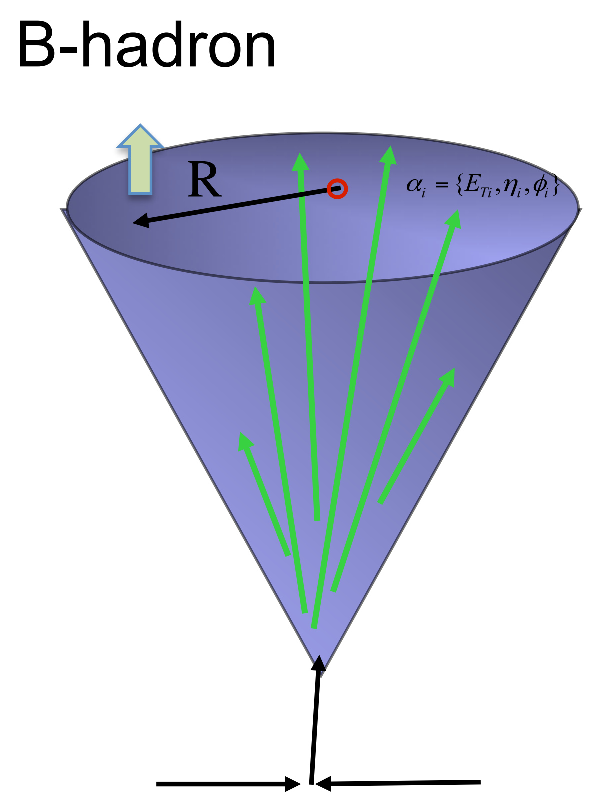


Figure 1: Ratio of gluon emission spectra off charm and light quarks for quark momenta $p_\perp = 10\,\mathrm{GeV}$ (solid line) and $p_\perp = 100\,\mathrm{GeV}$ (dashed); $x = \omega/p_\perp$.

B-jets in p+p collisions

- How to define a jet: need jet finding algorithms
  - kt algorithm, anti-kt algorithm, cone algorithm, ...



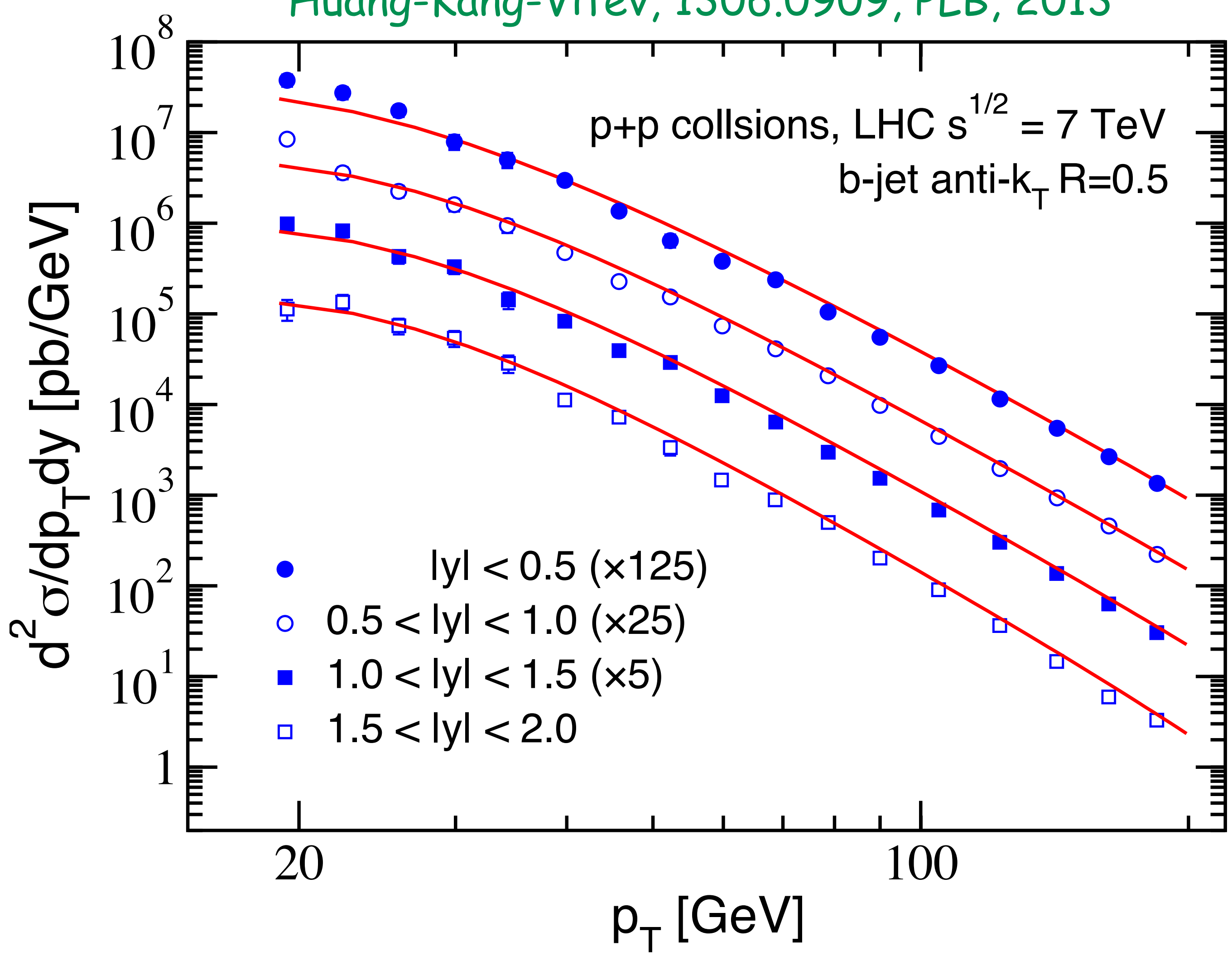
- Define b-jet
  - First find a jet. Next, with the jet radius parameter look for a B-hadron (b-quark for theory). Call it a b-jet … Or maybe require the b-quark to be leading … Or maybe some more creative substructure ("single b-quark jet" at Fermilab)
  - Note that the parent parton might have nothing to do with a b-quark

B-jets in p+p collisions

- No readily available NLO calculation for b-jet production (MC@NLO ...)
- PYTHIA 8 (LO+LL parton shower)
- SlowJet program with an anti-kt algorithm versus FastJet shown to give the same result
- Good description to the b-jet cross section as a function of pt and rapidity y

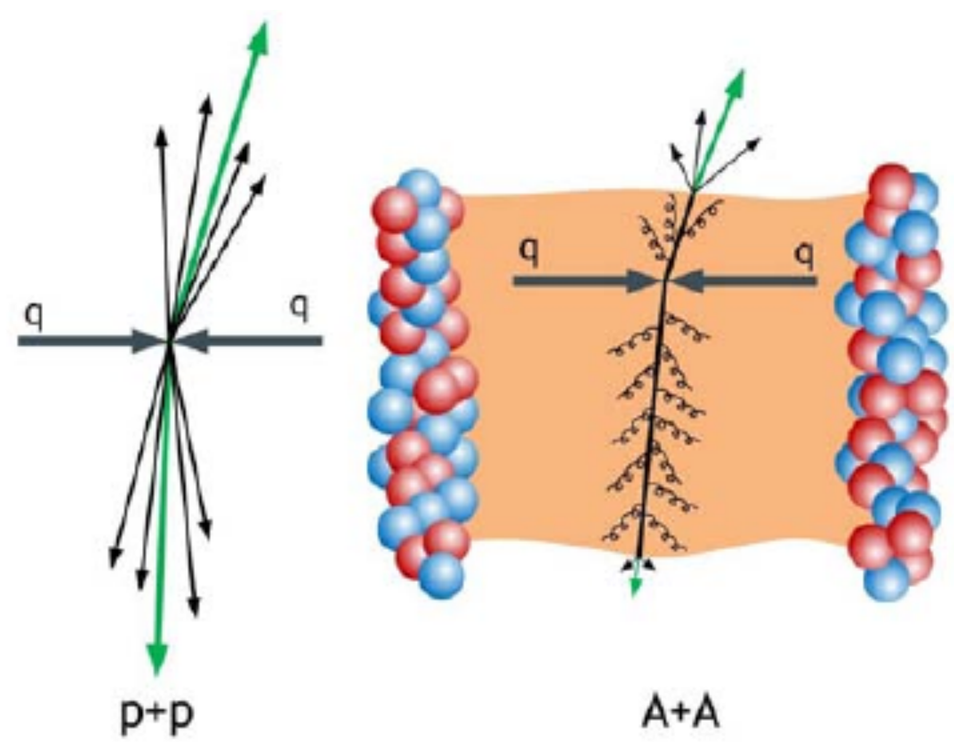
Hadronization corrections:
only important for pt < 30 GeV
and small jet radius R=0.2, 0.3

Huang-Kang-Vitev, 1306.0909, PLB, 2013



Hard partonic structure for b-jets

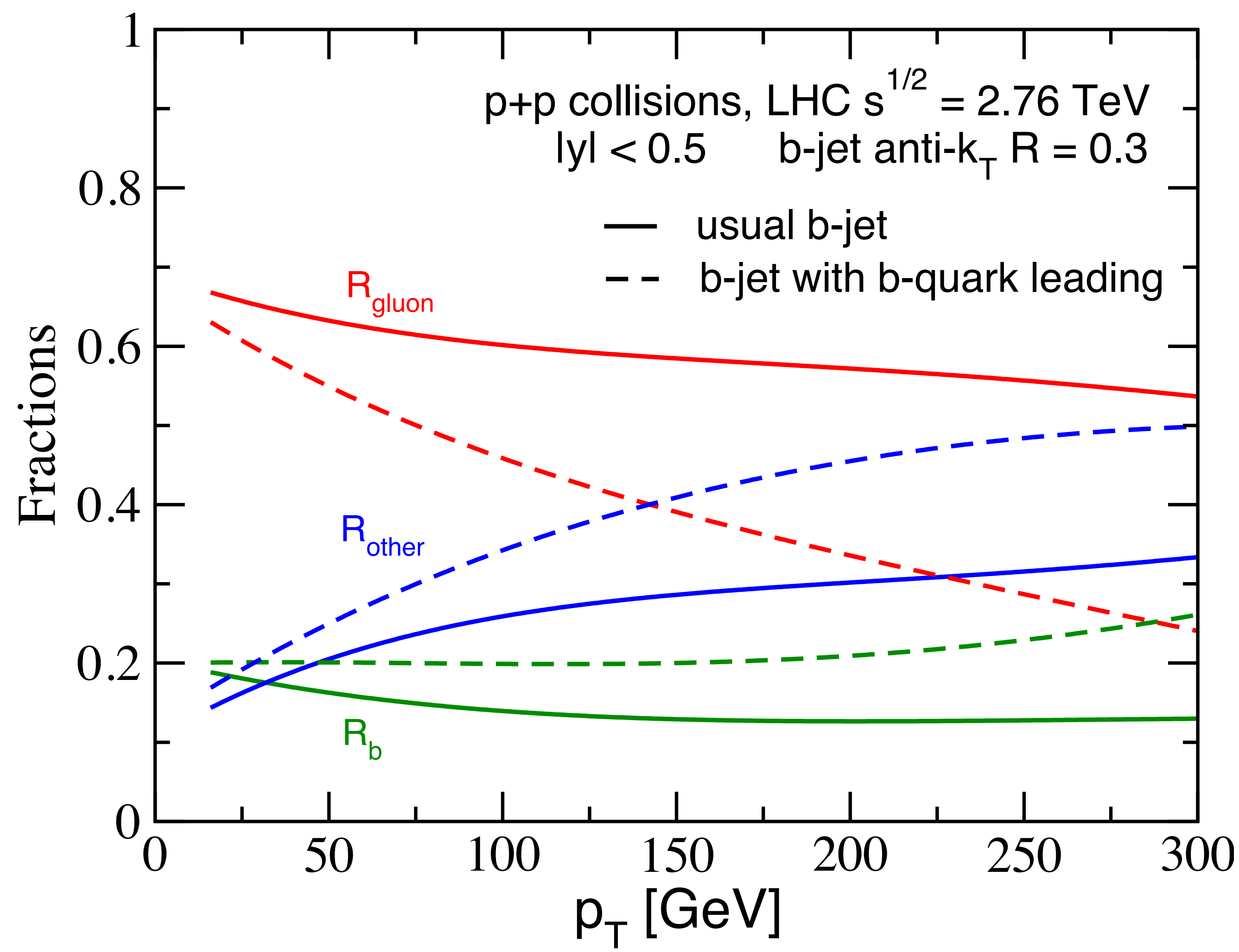
- Medium modification for b-jets in heavy ion collisions comes from both initial-state and final-state effects
  - Initial-state: cold nuclear matter (CNM) effects
  - Final-state: parton energy loss ⇒ have to understand the hard partonic structure for b-jets (whether light quark, gluon, or b quark)



Hard partonic structure for b-jets

- Simulation in Pythia

Huang-Kang-Vitev, 1306.0909, PLB, 2013



  - $R_{\rm gluon}$: fraction of $g \to b(\bar{b})$, i.e., hard process generates gluons, which then split into heavy quark pair as contained in b-jets (initiated by gluon)
  - $R_{\rm b-quark}$: fraction of $b(\bar{b}) \to b(\bar{b})$
  - $R_{\rm other}$: fraction of $q(\bar{q}) \to b(\bar{b})$
- A very small fraction of b-jets originate from a b-quark produced in the hard scattering

B-jet cross section calculation in heavy ion collisions

- Only a fraction of lost energy (medium induced parton shower) falls inside the cone, which can be computed as follows

$$f(R,\omega^{\rm coll})_{(s)} = \frac{\int_0^R dr \int_{\omega^{\rm coll}}^E d\omega \, \frac{\omega d^2 N^g_{(s)}}{d\omega dr}}{\int_0^{R^\infty} dr \int_0^E d\omega \, \frac{\omega d^2 N^g_{(s)}}{d\omega dr}}$$

  - (1 - f) is lost

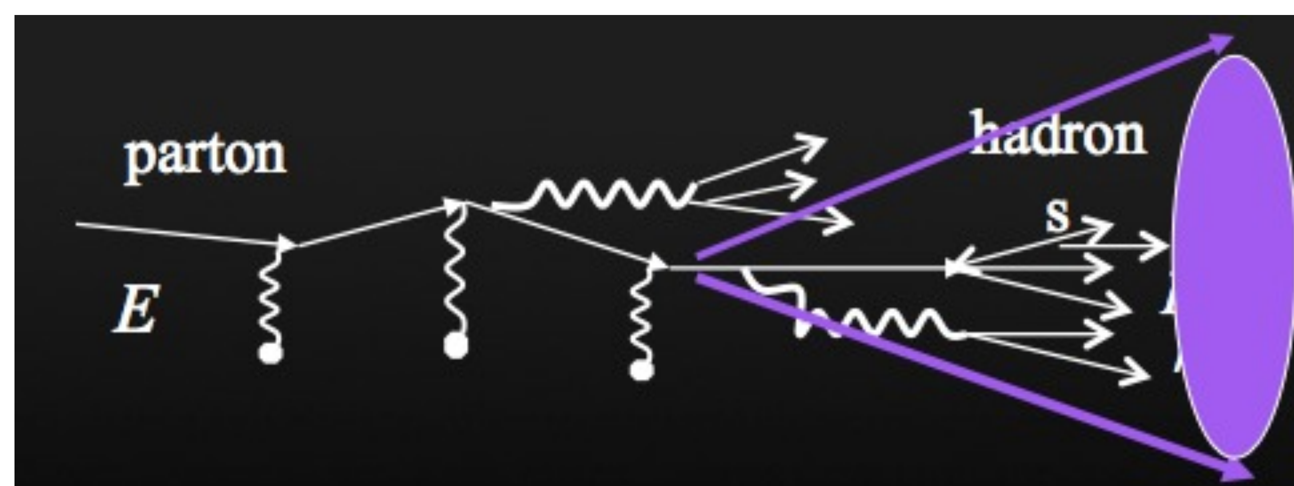
- In such a formalism, adjust $\omega^{\rm coll}$ such that

$$f(R^\infty,\omega^{\rm coll})_{(s)} = \Delta E^{\rm coll}/E$$

  - The right-hand side is simulated independently

- In order to get the jet with same energy, one has to start with a "higher" energy jet before the quenching

$$E_T' = E_T/(1-(1-f_{q,g})\cdot\epsilon)$$



B-jet cross section calculation in heavy ion collisions

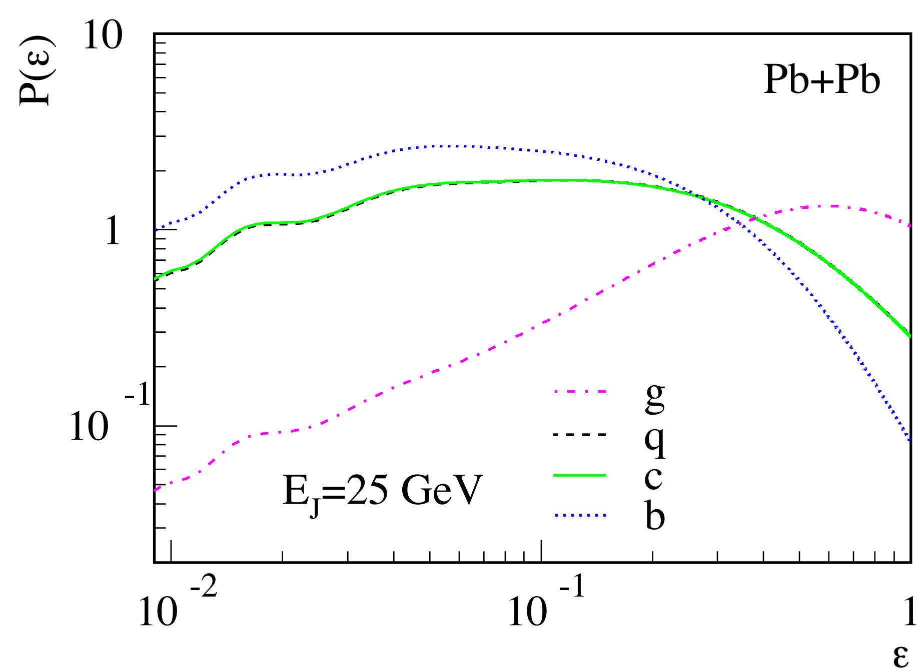
- Eventually the b-jet cross section is calculated as the following

$$\frac{1}{\langle N_{\rm bin}\rangle}\frac{d^2\sigma_{AA}^{\rm b-jet}(R)}{dy dp_T} = \sum_{(s)}\int_0^1 d\epsilon\, \frac{P_{(s)}(\epsilon)}{(1-[1-f(R,\omega^{\rm coll})_{(s)}]\epsilon)} \times \frac{d^2\sigma_{(s)}^{\rm CNM,LO+PS}\left(|J(\epsilon)|_{(s)}p_T\right)}{dy dp_T}\,. \quad (5)$$

- $P(\varepsilon)$: the probability to lose energy (with a fraction of ε) due to multiple gluon emission

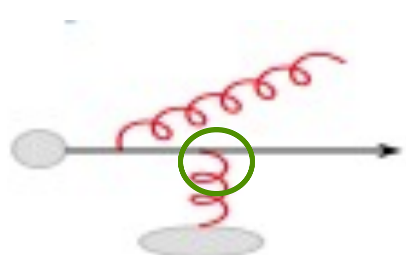
Kang-Vitev, 1106.1493, PRD, 2011

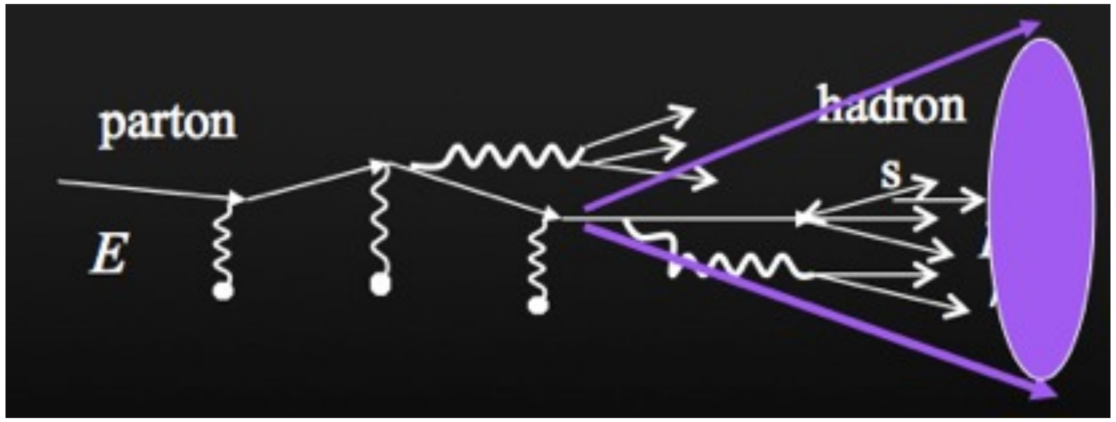
$$\int_0^1 P_{q,g}(\epsilon_i)d\epsilon_i = 1\,, \quad \int_0^1 \epsilon_i P_{q,g}(\epsilon_i)d\epsilon_i = \frac{\Delta E_{q,g\,i}}{E_i}$$

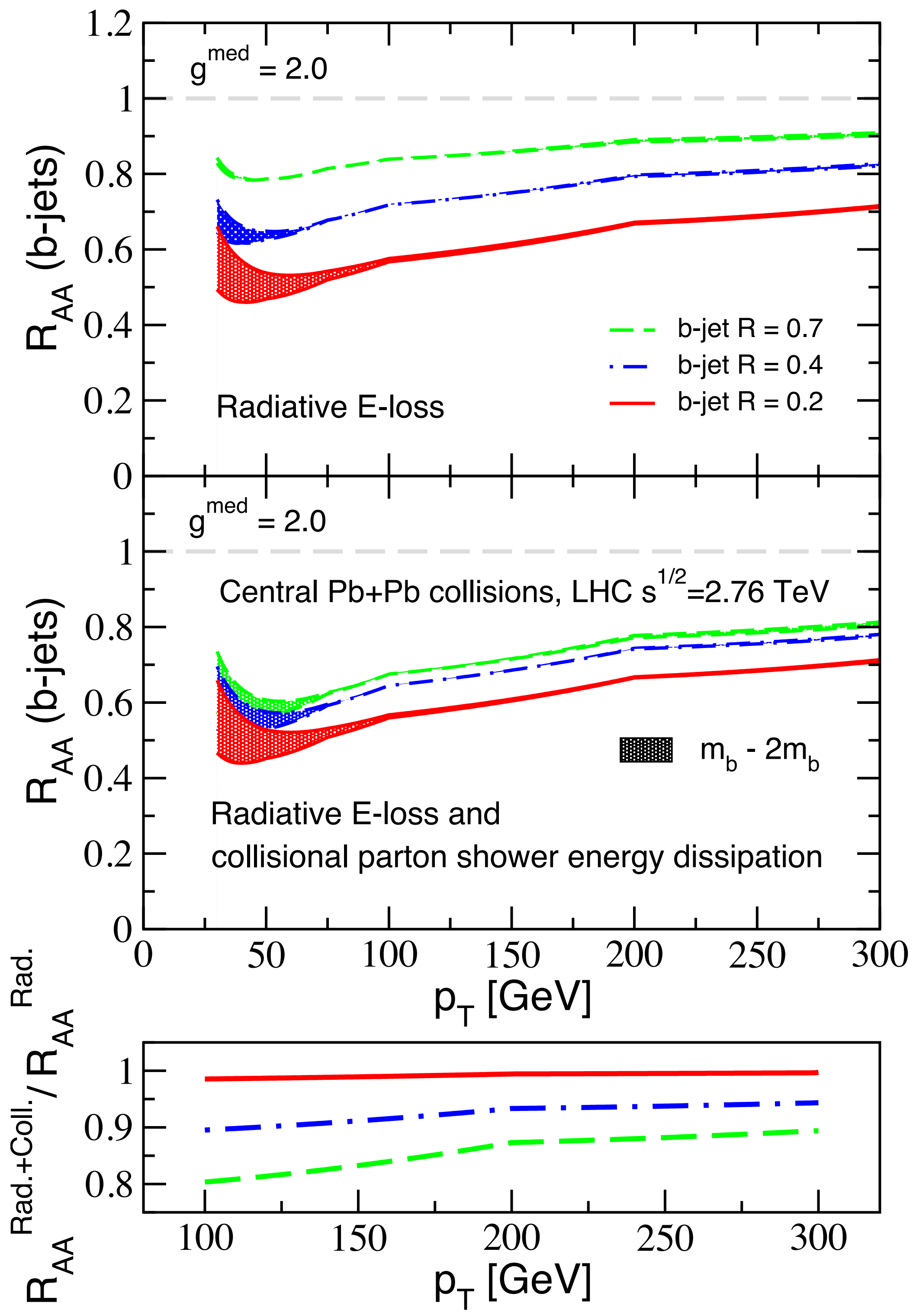


B-jet result at LHC 2.76 TeV Pb+Pb collisions

- General trend: smaller R leads to larger suppression, consistent with the intuition
- Radiative energy loss is larger for small cone size while collisional energy loss is less sensitive to the jet cone size





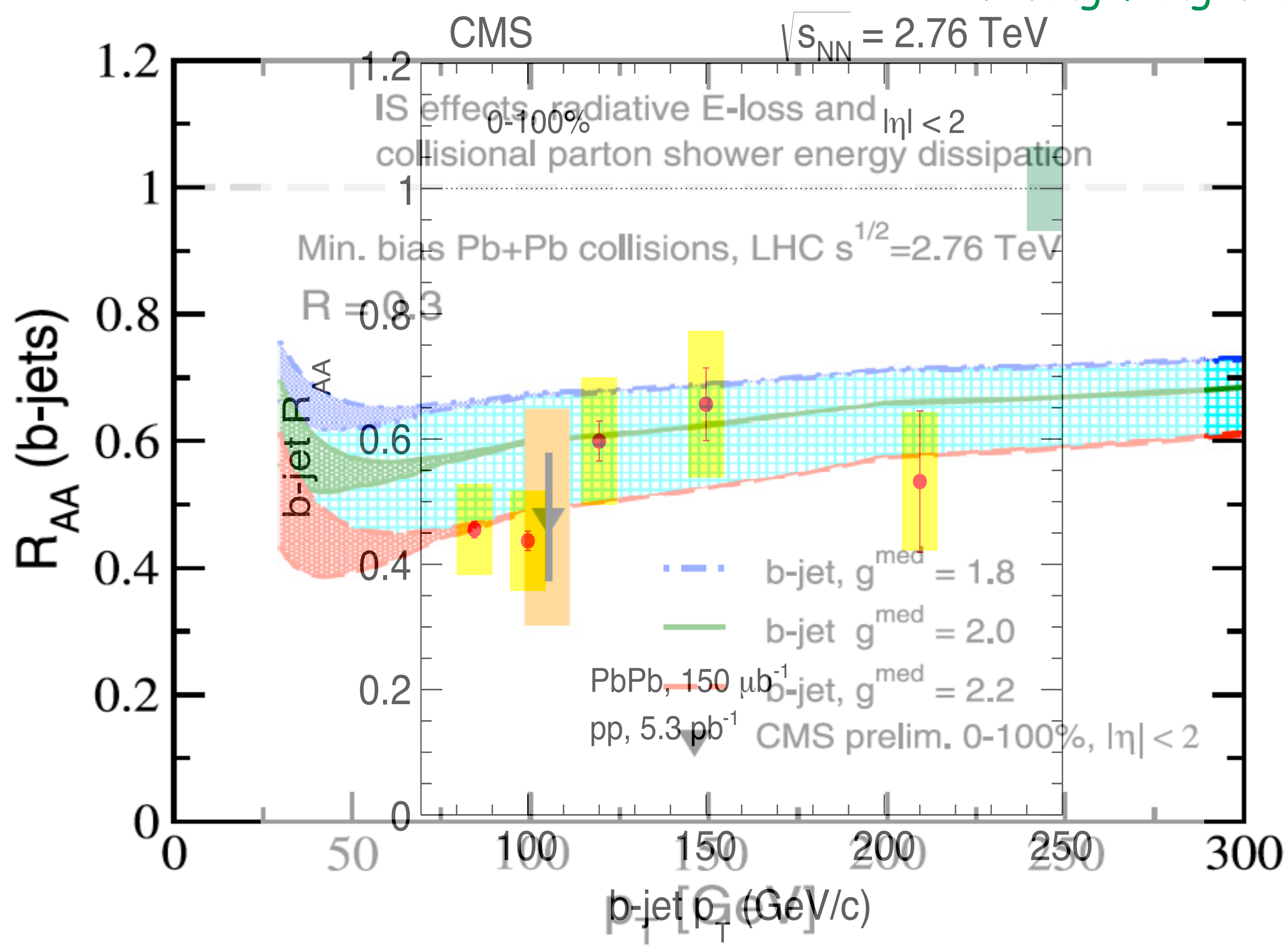


Huang-Kang-Vitev, 1306.0909, PLB, 2013

Works fine

- Compared with the most recent CMS b-jet data

Huang-Kang-Vitev, 1306.0909, PLB, 2013



- b-jet at high pt is not really sensitive to the b-quark energy loss
- Could it be the same as heavy flavor meson? (q, g can fragment equally)

Is standard "energy loss" the full story?

- Soft approximation
  - Under soft approximation, the parton does not change identity, so the energy loss has its true meaning
  - If the incoming quark loses 90% of its energy (through gluon radiation), the gluon has become the main content, which will fragment to the hadron

$$D_{h/c}(z) \Rightarrow \int_0^{1-z} d\epsilon\, P(\epsilon) \frac{1}{1-\epsilon} D_{h/c}\left(\frac{z}{1-\epsilon}\right)$$

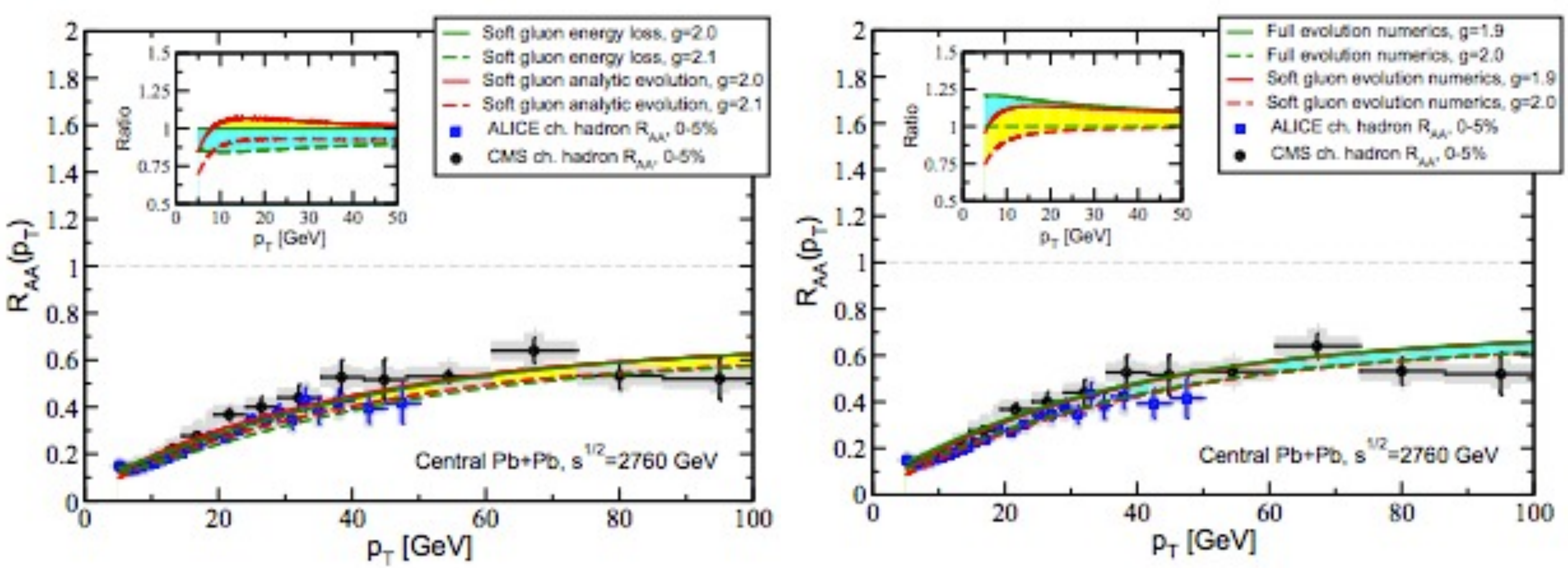
- Need full calculation contains the full "DGLAP-type" evolution (in medium), which can convert quark to gluon, and/or vice versa

A recent study: go beyond soft approximation

- DGLAP type evolution: splitting kernel in medium is derived from SCET$_G$, consistent with GLV for diagonal pieces (now with off-diagonal piece)

Kang-Ovanesyan-Vitev, et.al. 1405.2612

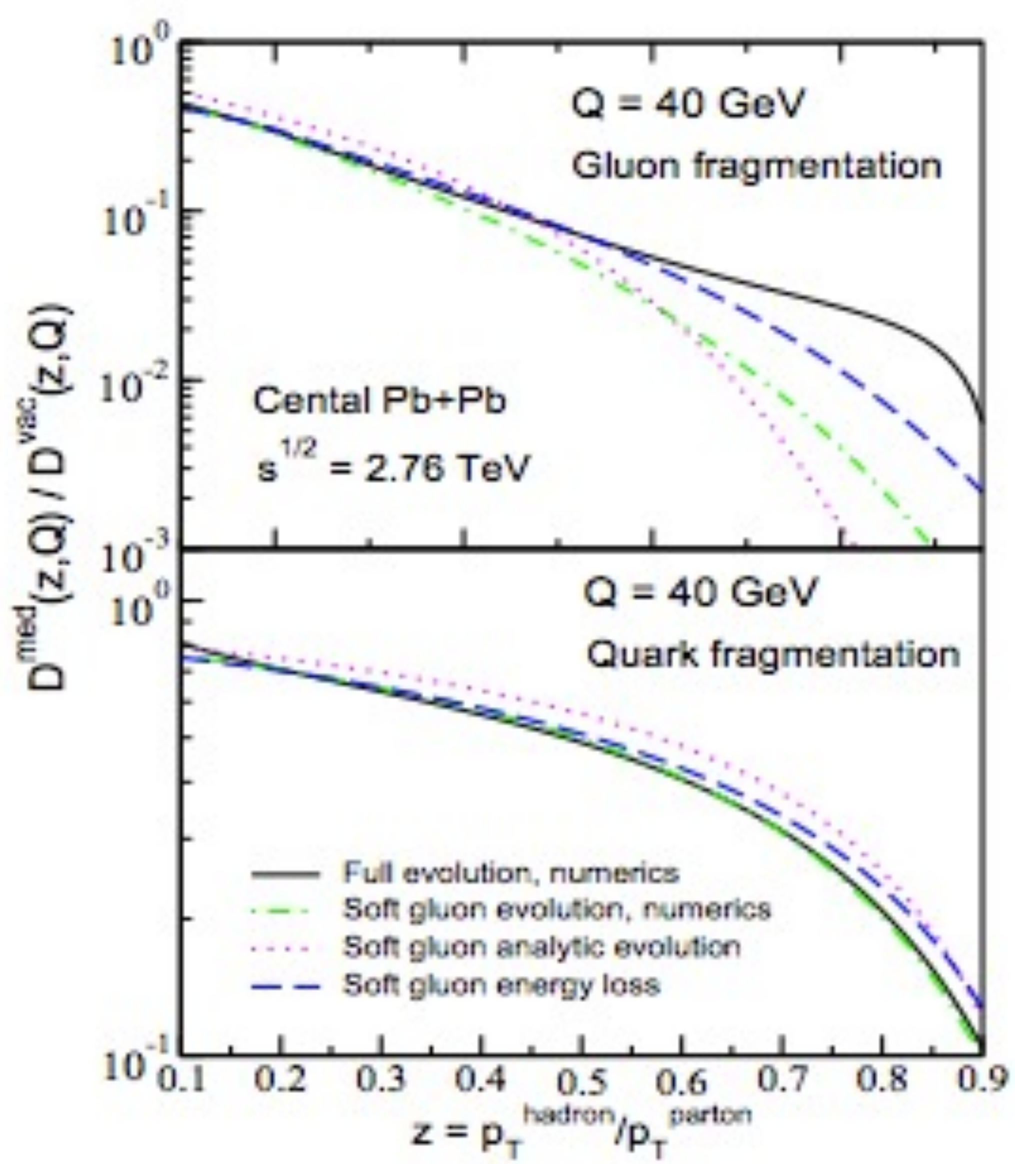
$$R_{AA}(p_T) = \frac{H(\mu, p_T) \otimes f(\mu) \otimes f(\mu) \otimes D^{\rm med}(\mu)}{H(\mu, p_T) \otimes f(\mu) \otimes f(\mu) \otimes D(\mu)}$$



The modified fragmentation function does vary

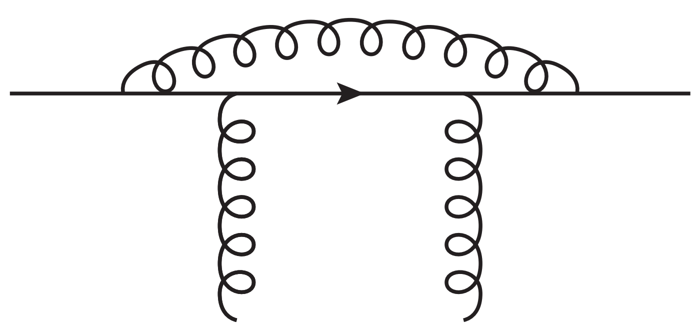
- Difference is pronounced at large z region for modified fragmentation function
- The observed hadron samples a wide range of z
- In the presence of QGP, biased toward lower values of z

Kang-Ovanesyan-Vitev, et.al. 1405.2612



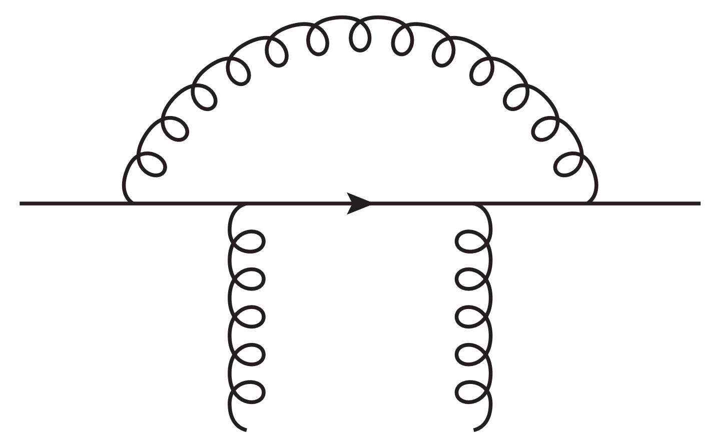
Is standard “energy loss” the full story?

- Collinear approximation
  - So far the above picture contains only the modification to the fragmentation function (modified evolution): come from the region where gluon is radiated collinear to the parent quark - collinear approximation?!
  - A complete NLO calculation to pt spectrum of course also has to include the NLO hard-part function: come from the region where gluon is radiated outside the collinear region



$\ell_T^2 < \mu^2$

Renormalized FFs



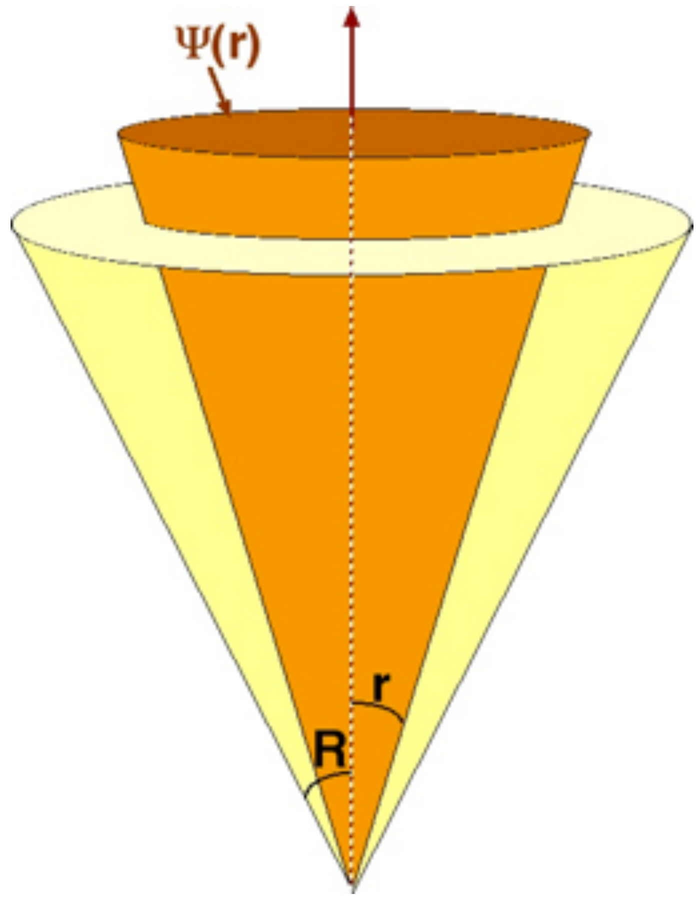
$\ell_T^2 > \mu^2$

NLO hard-part

Jet shape

- Jet shape gives the fraction of the total jet energy, with a jet having radius R, within radius r

$$\Psi(r;R) = \frac{\sum_{i<r} E_T^i}{\sum_{i<R} E_T^i} \qquad\qquad \Psi(r=R;R)=1$$



- Differential jet shape

$$\psi(r;R) = \frac{d\Psi(r;R)}{dr}$$

- Leading order jet gives a delta-function jet shape
  - Since the single parton is the jet, so jet has no internal structure
- Structure only happens at next-to-leading order for jet cross section
  - Will give leading order (first nontrivial order) jet shape

How to compute leading order jet shape

- The probability of final-state emission from a parton of type a is given by

$$dP_a = \sum_b \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\frac{d\phi}{2\pi} dz P_{a\to bc}(z)$$

- Jet shape at LO

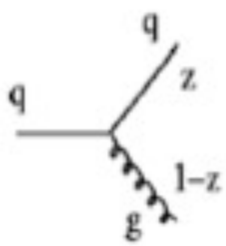
M. Seymour 1998

$$\psi_a(r;R) = \sum_b \frac{\alpha_s}{2\pi}\frac{2}{r}\int_0^{1-Z} dz z P_{a\to bc}(z)$$

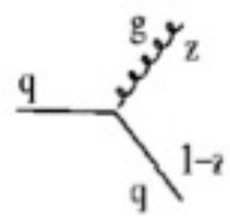
- Upper limit "Z" comes from the phase space limit: the requirements that both partons b, c be within R of the jet axis and the opening angle be less than $R_{sep}R$
  - Potential overlapping cones: introduce an adjustable parameter R_{sep}, whereby if two partons are within an angle $R_{sep}R$ of each other, they are merged into one jet

Splitting functions

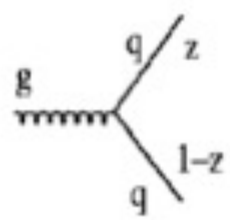
- Splitting functions are well-known from DGLAP equations



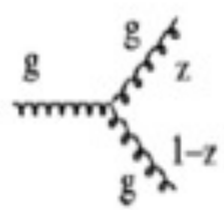
$$P_{qq}^{(1)}(x) = C_2(F)\left[(1+x^2)\left(\frac{1}{1-x}\right)_+ + \frac{3}{2}\delta(1-x)\right]$$



$$P_{gq}^{(1)}(x) = C_2(F)\,\frac{(1-x)^2+1}{x}$$



$$P_{qg}^{(1)}(x) = T(F)\left[(1-x)^2 + x^2\right]$$



$$P_{gg}^{(1)}(x) = 2C_2(A)\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \left(\frac{11}{6}C_2(A) - \frac{2}{3}T(F)n_f\right)\delta(1-x),$$

- LO jet shape for quark and gluon

M. Seymour 1998

$$\psi_q(r) = \frac{C_F\alpha_s}{2\pi}\frac{2}{r}\left(2\log\frac{1}{Z} - \frac{3}{2}(1-Z)^2\right)$$

$$\psi_g(r) = \frac{C_A\alpha_s}{2\pi}\frac{2}{r}\left(2\log\frac{1}{Z} - \left(\frac{11}{6} - \frac{Z}{3} + \frac{Z^2}{2}\right)(1-Z)^2\right) + \frac{T_R N_f\alpha_s}{2\pi}\frac{2}{r}\left(\frac{2}{3} - \frac{2Z}{3} + Z^2\right)(1-Z)^2$$

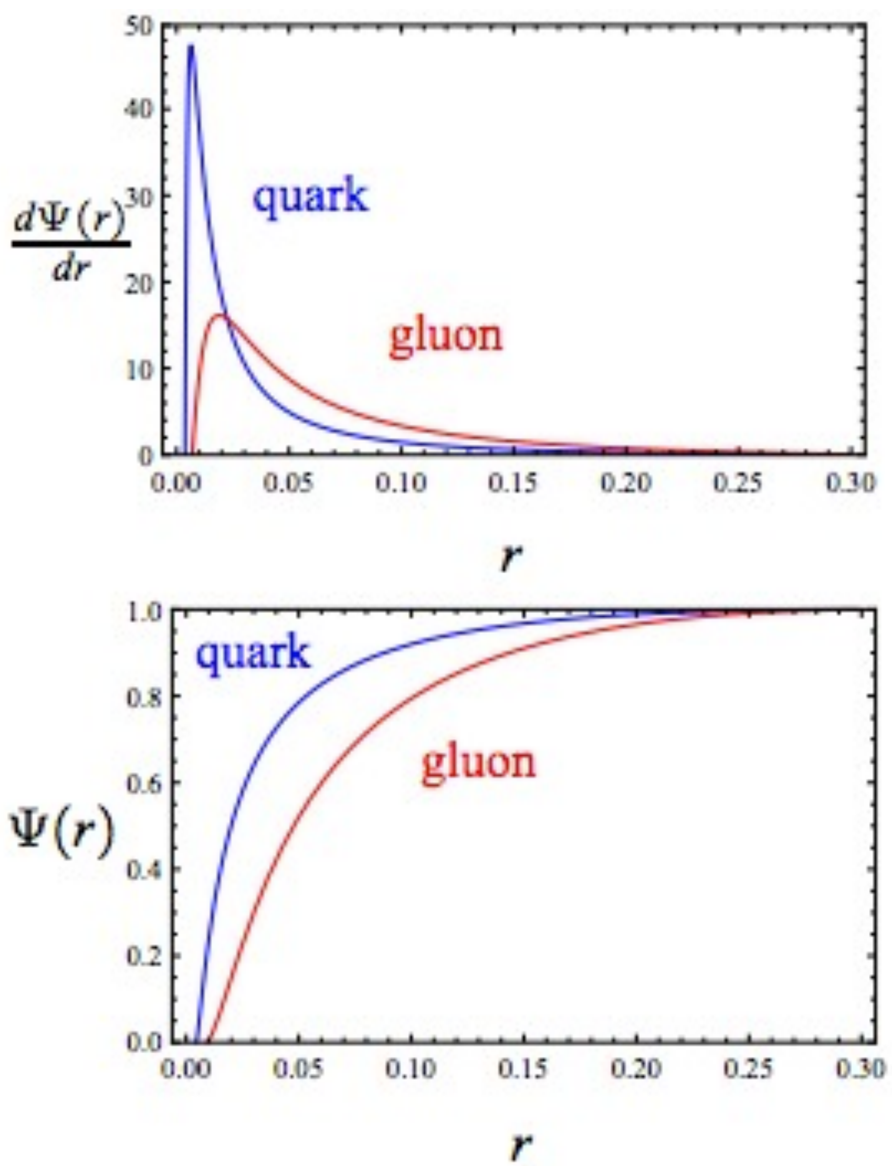
$$Z = \frac{r}{r+R} \text{ if } r < (R_{sep}-1)R$$
$$= \frac{r}{R_{sep}R} \text{ if } r > (R_{sep}-1)R$$

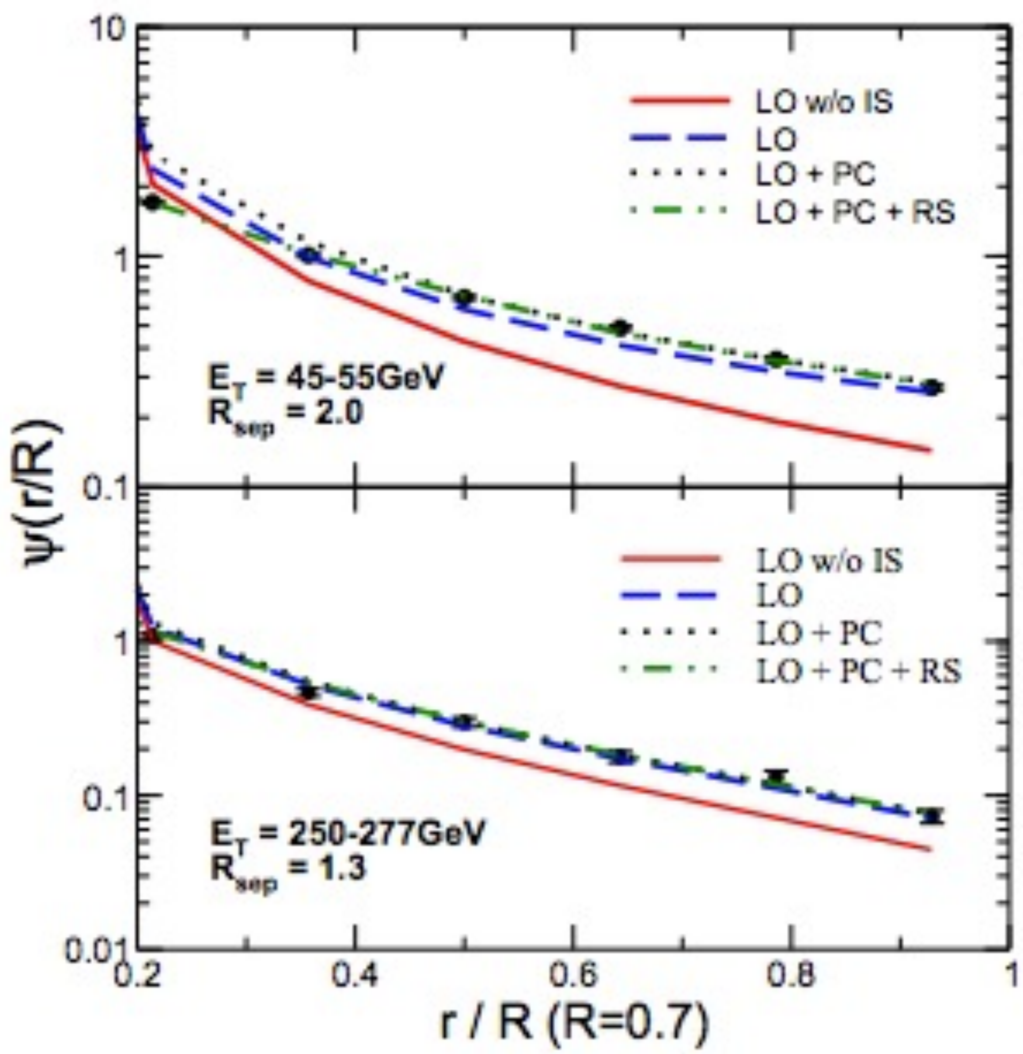
- Jet shape in the measurement: need also the quark and gluon jet fractions

$$\psi(r;E_T) = f_q(E_T,\sqrt{s})\psi_q(r;E_T) + f_g(E_T,\sqrt{s})\psi_g(r;E_T)$$

LO shape

- Quark jets are more localized than gluon jets
- There are more effects to add in: initial-state radiation (that happens to be inside the jet cone by chance), power corrections, ...

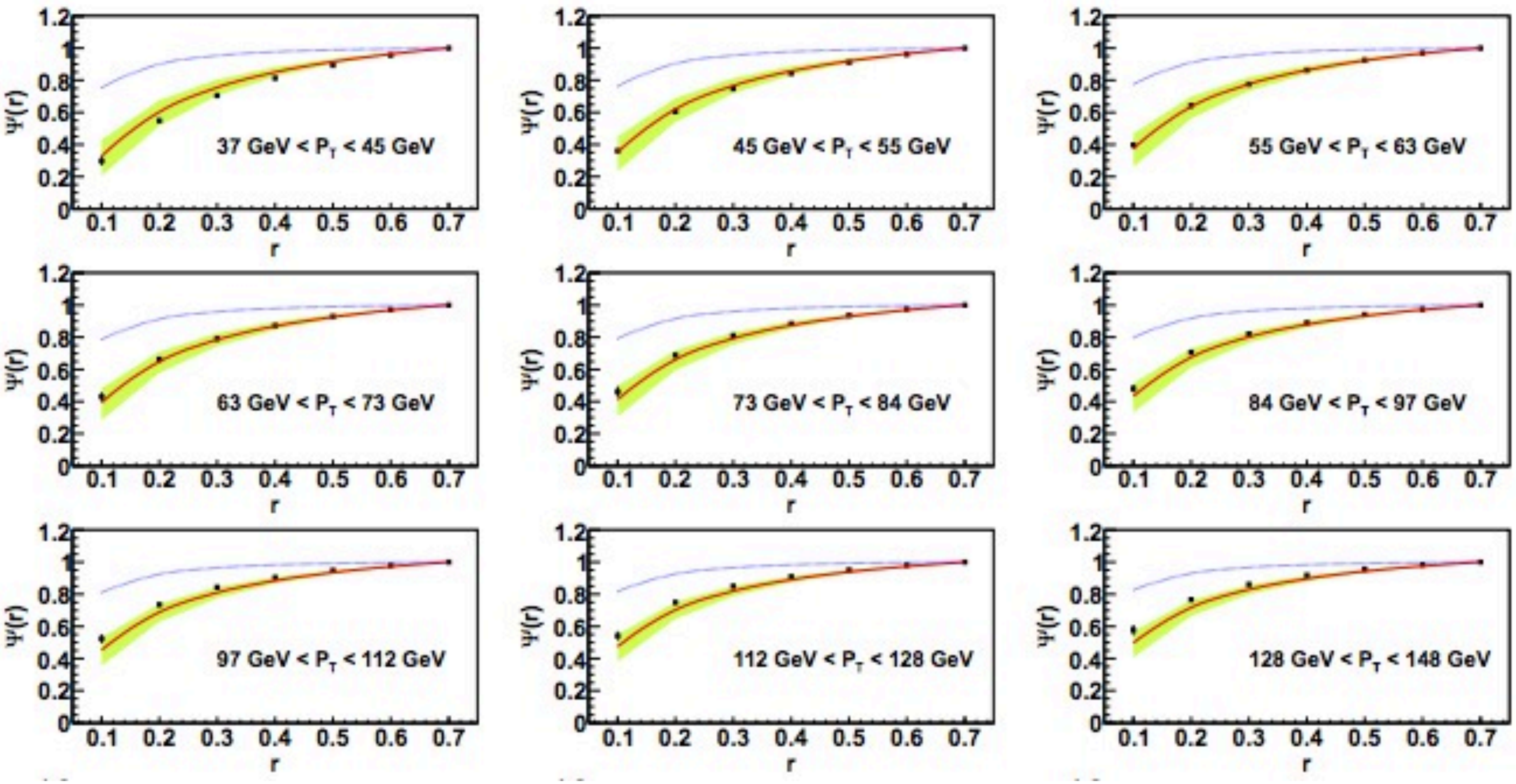




Resummation and etc

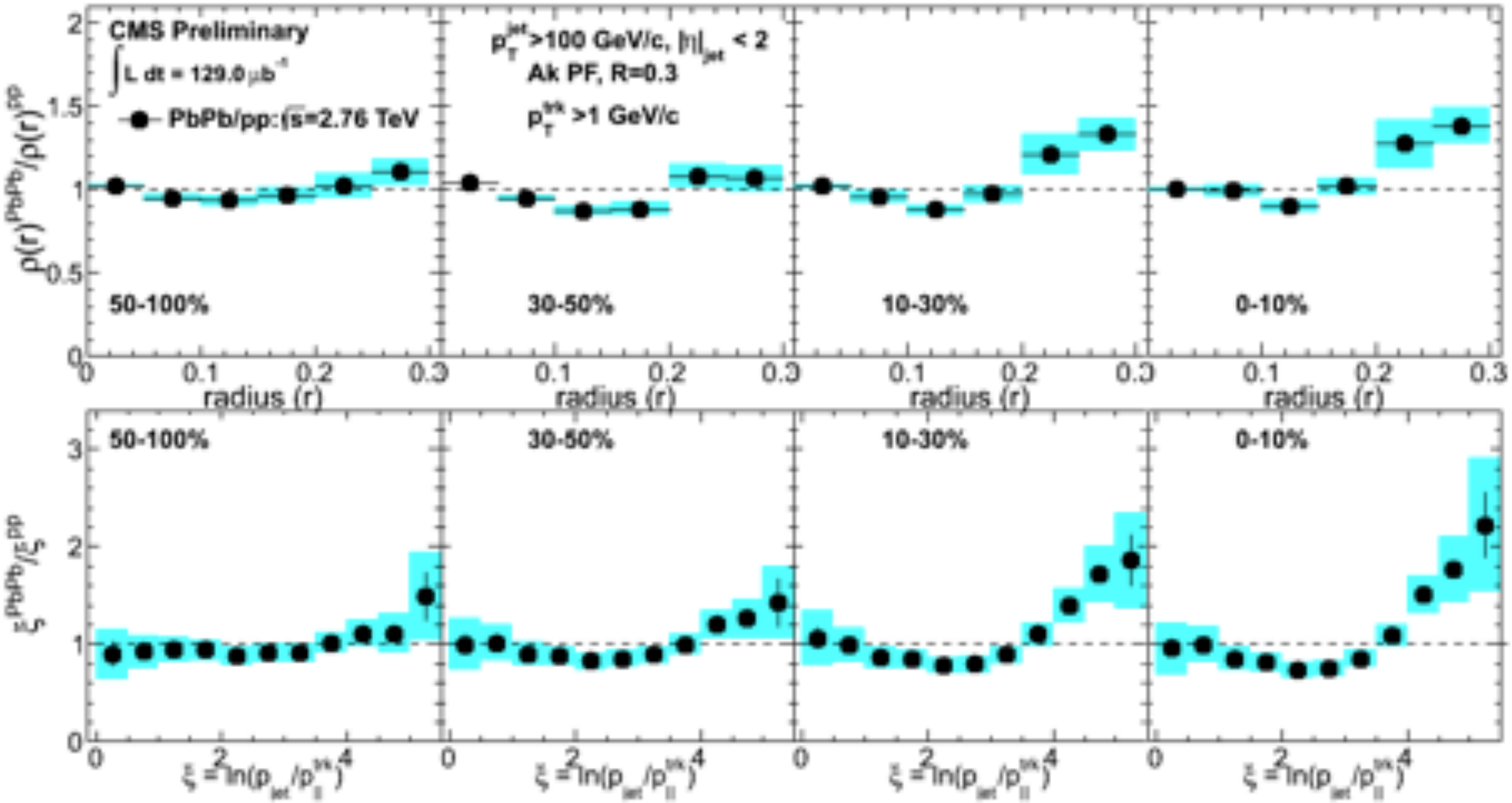
- Jet shape can be well controlled in pQCD
  - When r<<R, there could be large logarithms of $\alpha_s \ln^2(R/r)$

H.N. Li, Z. Li, C.P. Yuan 2011, 2013



Jet shape in heavy ion collisions

- In medium multiple scattering tends to broaden the jet distribution
  - Naively speaking, the small r part has suppression, large r part has enhancement



Summary

- Jets are certainly more powerful but also more complicated in terms of “analytical-type” computation
- Jet substructure hopefully could reveal more detailed dynamics about jet quenching, help us understand the underlying mechanism