Jet evolution in a dense QCD medium: III

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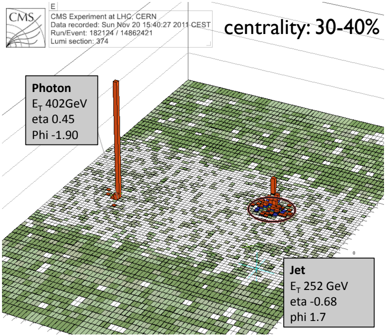
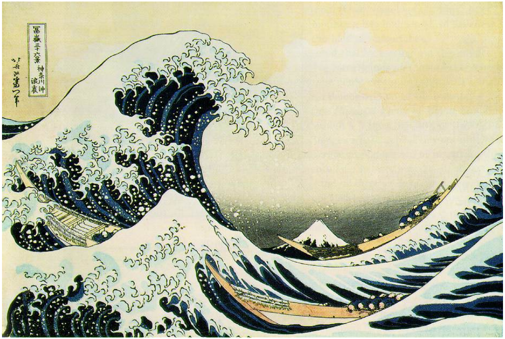
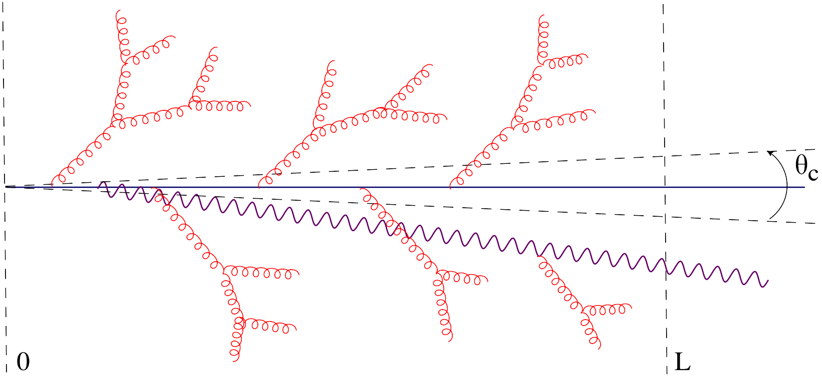




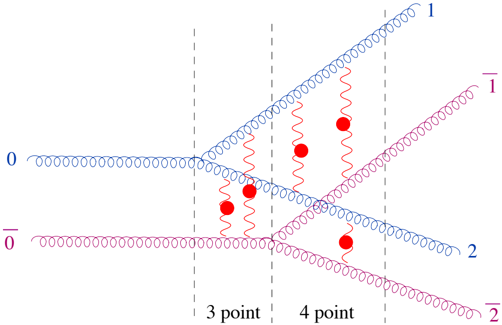
A typical gluon cascade



- The leading particle emits mostly soft gluons ($x \ll 1$)
- The subsequent branchings of these soft gluons are quasi–democratic
- Very efficient in transporting the energy at small x, or large angles

A few words on the formalism

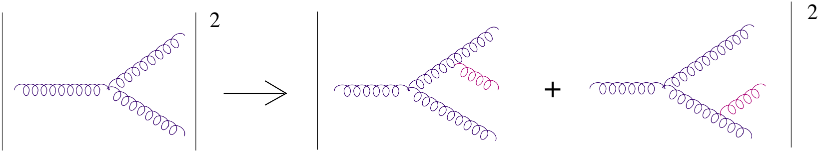
- $1 \to 2$ gluon branching: amplitude $\times$ complex conjugate amplitude



- 'Medium' = randomly distributed scattering centers (Gaussian)
  - Coulomb scattering with Debye screening
  - multiple scattering in eikonal approximation (one Wilson line per gluon)
- Cross–section: 3–p and 4–p functions of the Wilson lines
  - similar to the 2–p function (the 'dipole'), but more complicated

Interference effects

- The theory of multiple emissions can be quite complicated
- Successive quantum emissions are generally not independent
  - one sums the amplitudes, then one takes the modulus squared

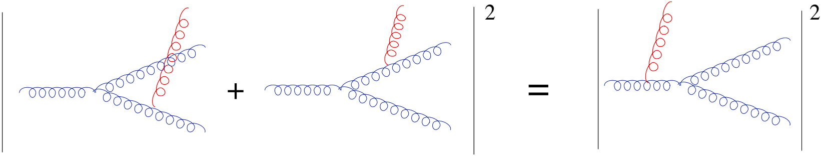


  - the 'cross–terms' represent interference effects

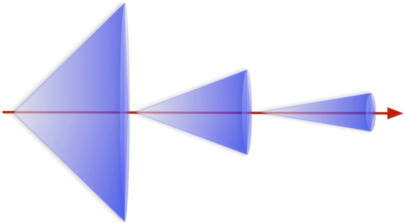
- During gluon formation, the sources must be coherent with each other
  - their overall color charge should not change until the next emission
  - obviously satisfied in the vacuum, but not also in the medium (color exchanges with the medium constituents)

Angular ordering in the vacuum

- A subsequent emission at large angles sees the overall color charge



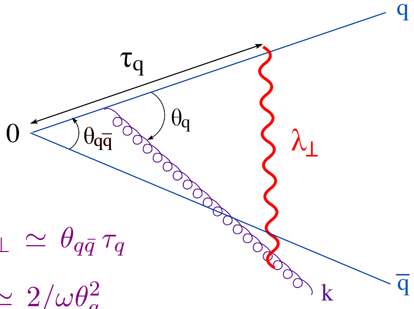
- Destructive interference effects leading to angular ordering



- An essential feature of jet fragmentation in the vacuum.

A ‘color antenna’ in the vacuum

- A $q\bar{q}$ pair in a color singlet state (say, as produced by the decay of a photon) which propagates at a fixed angle $\theta_{q\bar{q}}$
- During formation, the gluon must overlap with both sources



$$\lambda_\perp \simeq 2/k_\perp \gtrsim r_\perp \simeq \theta_{q\bar{q}}\,\tau_q$$

$$k_\perp \simeq \omega\,\theta_q\,, \quad \tau_q \simeq 2/\omega\theta_q^2$$

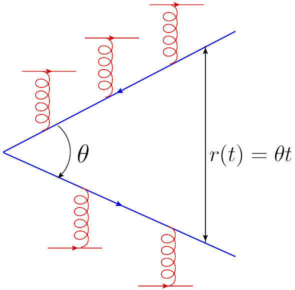
$\Longrightarrow \theta_q \gtrsim \theta_{q\bar{q}}$: large angle emission (out of cone)

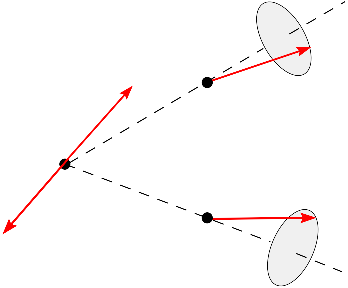
- Large–angle gluons see only the total color charge (here, zero)

$\Longrightarrow$ the out–of–cone radiation is washed out by interference

Color antenna in the medium

- The two quarks lose their color coherence via rescattering
- The instantaneous color state of each quark: the respective Wilson line
- Their color correlation is measured by the dipole S–matrix





$$S(\tau,\theta) \simeq \exp\left\{-\frac{1}{4}\hat{q}\int_0^{\tau} \mathrm{d}t\, r^2(t)\right\} \simeq \exp\left\{-\frac{1}{12}\hat{q}\theta^2\tau^3\right\}$$

- Color coherence is lost for $\tau \gtrsim \tau_{\rm coh} \sim 1/(\hat{q}\theta^2)^{1/3}$

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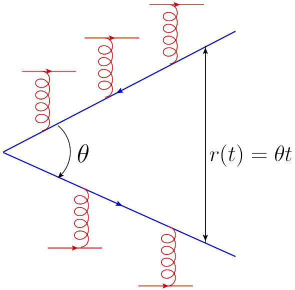
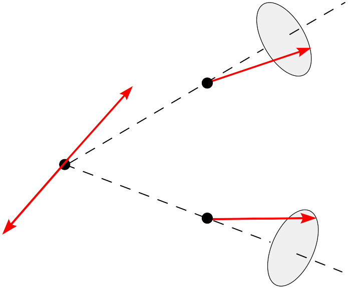


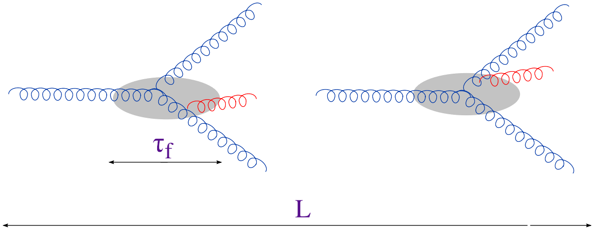


- If the antenna is generated via gluon splitting: $\theta = \theta_f(\omega) \sim (\hat{q}/\omega^3)^{1/4}$

$$\tau_{\rm coh} \sim \frac{1}{(\hat{q}\theta_f^2)^{1/3}} \sim \sqrt{\frac{2\omega}{\hat{q}}} = \tau_f(\omega)$$

Multiple emissions : medium

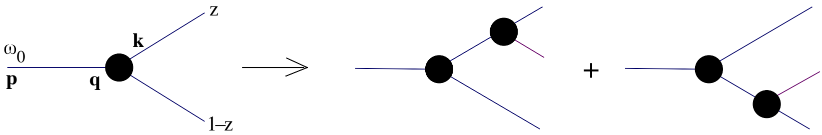
- In medium, color coherence is rapidly lost via rescattering
  Mehtar-Tani, Salgado, Tywoniuk (arXiv: 1009.2965; 1102.4317); Casalderrey-Solana, E. I. (arXiv: 1106.3864)



- The interference effects are suppressed by a factor $\tau_f/L \ll 1$
  $\triangleright$ the respective phase–space is proportional to τ_f, instead of L
  Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)
- Successive emissions of soft gluons ($\omega \ll \omega_c$) can be treated as independent (no interference, no angular ordering)

A classical branching process

- Medium–induced jet evolution $\approx$ a classical branching process



  - the $g \to gg$ splitting vertex (the 'blob') : the BDMPSZ spectrum
  - the propagator (the 'line') : transverse momentum broadening

- Markovian process in $D = 3+1$: ω, $\boldsymbol{k}_\perp$, time t (with $t \le L$)

  Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv:1311.5823)

- Well suited for Monte Carlo simulations

- The inclusive one–gluon distribution :

$$D(\omega, \boldsymbol{k}, t) \equiv \omega \frac{\mathrm{d}N}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}}$$

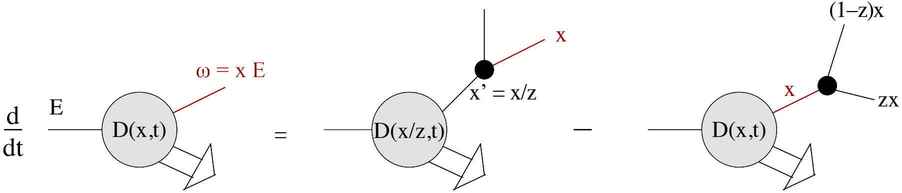
- Here, I will restrict myself to the 1+1 process involving $(\omega,\, t)$

The rate equation (1)

- Evolution equation for the gluon spectrum (energy per unit x) :

$$D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x} \quad \text{where} \quad x = \frac{\omega}{E} \quad \text{(energy fraction)}$$

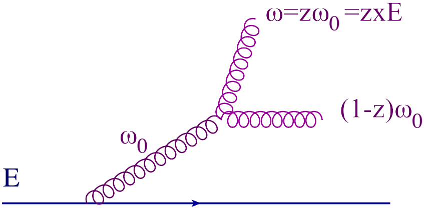
- $t \to t + \mathrm{d}t$: one additional branching with splitting fraction z



- Described by a rate equation : $\partial D/\partial t$ = Gain - Loss
  - 'Gain' : a gluon with fraction x is produced via the decay of a parent gluon with fraction $x' = x/z$
  - 'Loss': the gluon x decays into the gluon pair $(zx, (1-z)x)$, with any z

The BDMPSZ splitting rate

- Probability $\mathrm{d}P$ for a parent particle $\omega_0 = xE$ to split in a time $\mathrm{d}t$



$$\mathrm{d}P \simeq \alpha_s \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}t}{\tau_f(\omega)}$$

$$\simeq \alpha_s \frac{\mathrm{d}z}{z} \sqrt{\frac{\hat{q}}{zxE}}\,\mathrm{d}t$$

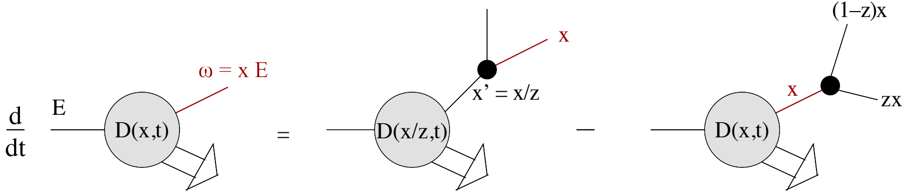
$$\mathcal{K}(z,x) \equiv \frac{\mathrm{d}P}{\mathrm{d}z\mathrm{d}t} \simeq \alpha_s \frac{1}{z} \sqrt{\frac{\hat{q}}{zxE}}$$

▷ parametric estimate correct when $z \ll 1$

- The general expression is symmetric under $z \to 1-z$:

$$\mathcal{K}(z,x) = \alpha_s \frac{P_{gg}(z)}{2\pi} \sqrt{\frac{\hat{q}}{z(1-z)xE}}\,, \qquad P_{gg}(z) \equiv N_c \frac{[1-z(1-z)]^2}{z(1-z)}$$

The rate equation (2)



$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z \left[2\mathcal{K}\left(z, \frac{x}{z}\right) D\left(\frac{x}{z}, t\right) - \mathcal{K}(z,x)\, D(x,t) \right]$$

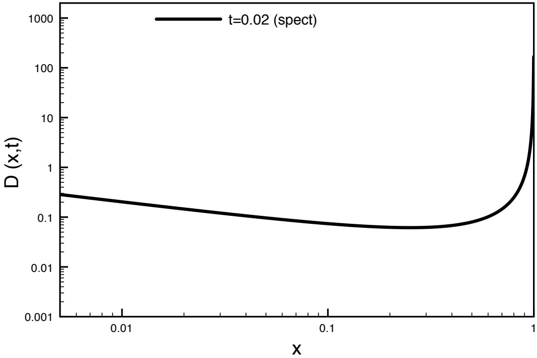
$$\mathcal{K}(z,x) \equiv \frac{\mathrm{d}P}{\mathrm{d}z\mathrm{d}\tau} = \frac{1}{2\sqrt{x}} \frac{1}{[z(1-z)]^{3/2}}, \qquad \tau \equiv \frac{\alpha_s N_c}{\pi}\sqrt{\frac{\hat{q}}{E}}\, t$$

- Previously conjectured and used for phenomenological studies :
  Baier, Mueller, Schiff, Son '01; AMY, '03; Jeon, Moore '05; MARTINI
- Carefully derived and studied (including exact solutions) in
  J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

First iteration

- Initial condition: $D(x,\tau=0)=\delta(x-1)$ (the leading particle)
- At small times: one iteration $\Rightarrow$ a single branching :

$$D^{(1)}(x,\tau)\;=\;2x\mathcal{K}(x,1)\tau\;=\;\frac{\tau}{\sqrt{x}(1-x)^{3/2}}$$

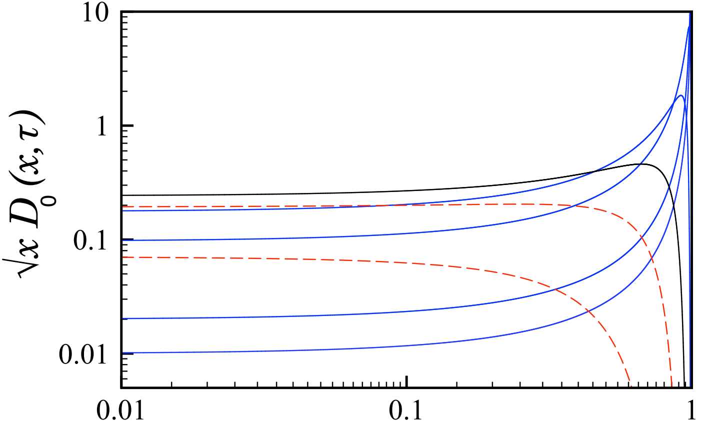


- This is the BDMPSZ spectrum : $D^{(1)}(x\ll 1,\tau)\simeq \tau/\sqrt{x}$

The spectrum with multiple branchings

- The spectrum at later times : exact solution to the rate equation

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}$$

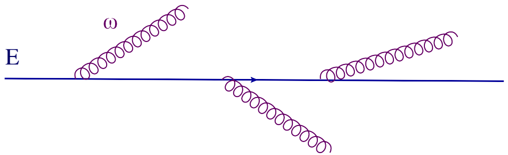


The leading particle

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}$$

- Correct initial condition: $D(x,\tau=0)=\delta(x-1)$
- Small times $\pi\tau^2 \ll 1$: a pronounced peak near $x=1$
  - the width of this peak : $1-x \sim \pi\tau^2 \sim \alpha_s^2\,\frac{\omega_c}{E}$
  - ... is associated with the emission of very soft gluons :

$$\omega_{\rm rad} \;=\; (1-x)E \;\lesssim\; \alpha_s^2\,\omega_c$$



- This peak (the 'leading particle') disappears when $\pi\tau^2 \sim 1$

The leading particle

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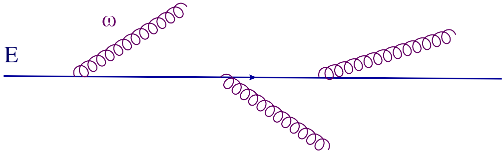

- Vice versa, a leading particle with sufficiently high energy $(E\gg\alpha_s^2\omega_c)$, survives in the final state

Quenching of hadron spectra : $R_{\mathrm{A+A}}$

$$\frac{\mathrm{d}\sigma^{\mathrm{med}}(E)}{\mathrm{d}E} = \int \mathrm{d}\epsilon\, \mathcal{P}(\epsilon)\, \frac{\mathrm{d}\sigma^{\mathrm{vac}}(E+\epsilon)}{\mathrm{d}E}$$

- $\mathcal{P}(\epsilon)$ with $\epsilon \ll E$: probability density for a 'leading particle' with initial energy $E+\epsilon$ to lose a small amount ϵ

$$\mathcal{P}(\epsilon) = \frac{\mathrm{d}N}{\mathrm{d}\omega}\Big|_{\omega=E} = \frac{1}{E}\, D\big(x, \tau_L\big)\Big|_{x=\frac{E}{E+\epsilon}}$$

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$$\mathcal{P}(\epsilon) = \bar{\alpha}\sqrt{\frac{2\omega_c}{\epsilon^3}}\, \exp\left\{-2\pi\bar{\alpha}^2\frac{\omega_c}{\epsilon}\right\}$$

▷ integral dominated by relatively soft values $\epsilon \sim \pi\bar{\alpha}^2\omega_c \ll \omega_c$

- Even $R_{\mathrm{A+A}}$ is controlled by the typical emissions of primary gluons, which are soft, and not by the average energy loss $\Delta E \sim \bar{\alpha}\omega_c$

- Fits to the data $\Rightarrow$ $\omega_c \simeq 50$ GeV, $L \simeq 5$ fm, $\hat{q} \sim 1 \div 2$ GeV^2/fm
(JET Collaboration, http://arxiv.org/pdf/1312.5003.pdf)

▷ the typical energy loss of the primary gluons: $\epsilon \sim \bar{\alpha}^2\omega_c \sim 5$ GeV

The scaling spectrum

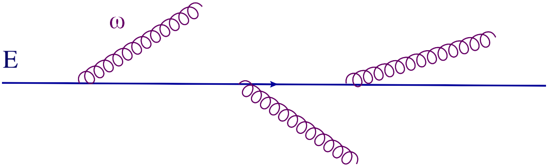
$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}$$

- Soft region of the spectrum $x \ll 1$ (and for any time τ) :

$$D(x \ll 1,\tau) \;\simeq\; \frac{\tau}{\sqrt{x}}\;\mathrm{e}^{-\pi\tau^2} \qquad \text{('scaling spectrum')}$$

  - formally : 'BDMPSZ × survival probability for the leading particle'

- This result at small x seems consistent with the following picture:



  - direct emissions by the leading particle ('all gluons are primary gluons')

The scaling spectrum

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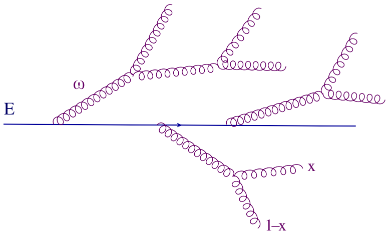
- ... but in reality one was expecting the following picture !



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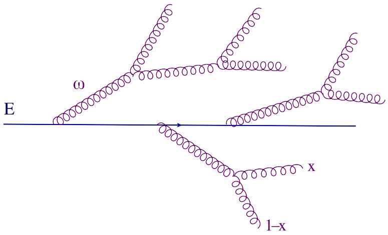
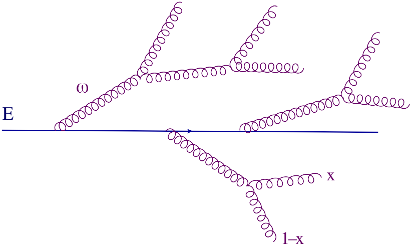


- The second picture is nevertheless the right one !

The fixed point

- Multiple branchings are very efficient at small $x \lesssim \tau^2$, yet they have no consequence on the shape of the spectrum



- The scaling spectrum is a fixed point of the rate equation:
  - precise cancellation between 'gain' and 'loss' terms at any $x \ll 1$
- Via successive branchings, the energy flows from large x to small x, without accumulating at any intermediate value of x

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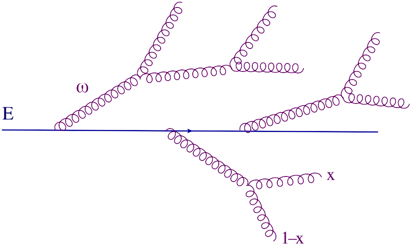


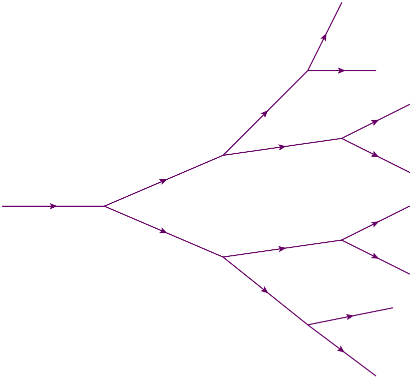
- the energy fraction which remains in the spectrum at time τ :

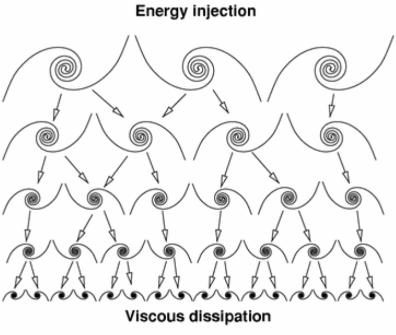
$$\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2}$$

- The scaling spectrum is a fixed point of the rate equation:
  - precise cancellation between 'gain' and 'loss' terms at any $x \ll 1$
- Via successive branchings, the energy flows from large x to small x, without accumulating at any intermediate value of x
- The energy flows out from the spectrum ... exponentially fast !

Wave turbulence

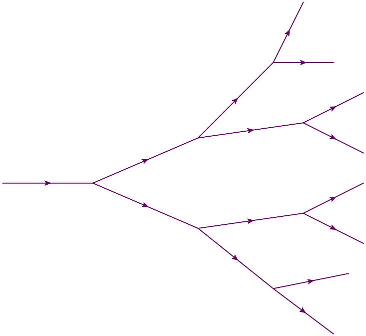
- The rate for energy transfer from one parton generation to the next one is independent of the generation (i.e. of x)



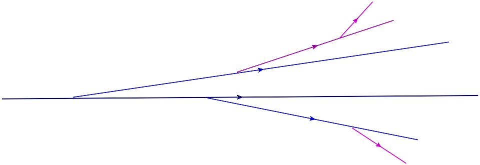


- The definition of wave turbulence *(Kolmogorov, '41; Zakharov, '92 ...)*
  - the prototype: Richardson cascade for breaking-up vortices

Compare to DGLAP cascade (jet in the vacuum)



in–medium cascade



DGLAP cascade

$\tau = \ln Q^2$ ('virtuality')

- No flow: the energy remains in the spectrum: $\int_0^1 \mathrm{d}x\, D(x,\tau) = 1$
- The asymmetric splittings amplify the number of gluons at small x
- Yet, the energy remains in the few partons with larger values of x
- That is, the energy remains at small angles

The energy flow

- Via successive branchings, the energy flows down to $x = 0$
  - formally, it accumulates into a 'condensate' at $x = 0$
  - physically, it goes below $x_{\rm th} = T/E \ll 1$, meaning it thermalizes
- The energy fraction carried away by this flow :

$$\mathcal{E}_{\rm flow}(\tau) \,\equiv\, 1 - \int_0^1 {\rm d}x\, D(x,\tau) \,=\, 1 - {\rm e}^{-\pi\tau^2}$$

- This energy emerges at very (arbitrarily) large angles !
- Universality : the energy loss via flow is
  - independent of the jet angular opening
  - independent of the details of the thermalization mechanism
  - a property of the gluon cascade, not of the in–medium dissipation

Applications to phenomenology

- In principle, the whole energy of the jet can be lost via flow
  - via arbitrarily soft particles which propagate at arbitrarily large angles

$$\mathcal{E}_{\rm flow}(\tau) \equiv 1 - \int_0^1 {\rm d}x\, D(x,\tau) = 1 - {\rm e}^{-\pi\tau^2}$$

- In practice, this depends upon the maximal value of τ, that is
  - upon the jet energy E & upon the medium properties $\hat{q}$ and L

$$\tau = \alpha_s \sqrt{\frac{\hat{q}}{E}}\, L \sim 0.3 \quad \text{for} \quad E = 100\ \text{GeV}$$

  - $1 - {\rm e}^{-\pi\tau^2} \sim 0.25 \Rightarrow$ about 25% of the energy is lost at large angles

- After restoring the physical units :

$$E_{\rm flow} \simeq \pi\, \alpha_s^2\, \hat{q} L^2 \qquad (\sim 20\,\text{GeV for } L = 5\text{ fm})$$

  - ... which is independent of the original energy E !

Energy loss at large angles

- How much of the jet energy emerges at angles $\theta > \theta_0$?

$$\theta(\omega) \simeq \frac{Q_s}{\omega} = \frac{Q_s}{xE} \implies \theta(x) > \theta_0 \longleftrightarrow x < x_0 \equiv \frac{Q_s}{E\theta_0}$$

- The energy fraction at $x < x_0$ has 2 components: 'spectrum' & 'flow'

$$\mathcal{E}(x \leq x_0, \tau) = \int_0^{x_0} \mathrm{d}x \, D(x, \tau) + \mathcal{E}_{\text{flow}}(\tau)$$

  - the 'spectrum' piece is dominated by the relatively hard gluons with $x \sim x_0$
  - the 'flow' piece is independent of x_0 and dominated by very soft gluons with $x \sim x_{\text{th}} \ll x_0$
  - 'flow'dominates over 'spectrum' for sufficiently large angle θ_0

- Di–jet asymmetry (energy loss at large angles) is controlled by flow

Energy loss at large angles

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- The energy fraction at $x < x_0$ has 2 components: 'spectrum' & 'flow'

$$\mathcal{E}(x \leq x_0, \tau) = 2\tau\sqrt{x_0}\,\mathrm{e}^{-\pi\tau^2} + (1 - \mathrm{e}^{-\pi\tau^2}) \simeq 2\tau\sqrt{x_0} + \pi\tau^2$$

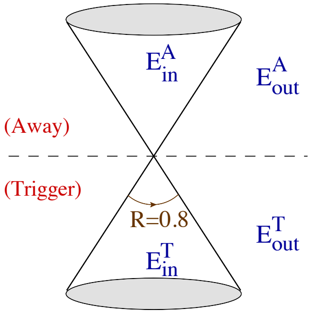
  - we assume $x_{\rm th} < x_0 \ll 1$ and $\pi\tau^2 \ll 1$
  - $E = 100$ GeV, $Q_s = 2.5$ GeV, $T = 1$ GeV, $\theta_0 = 0.5$

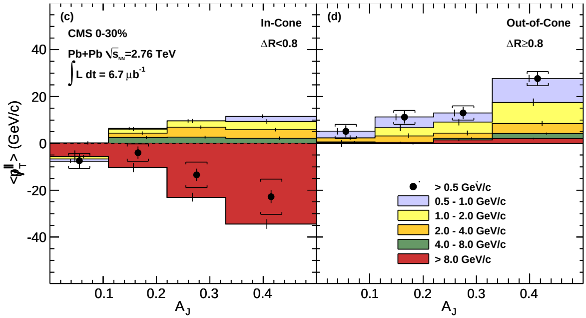
    $\implies x_{\rm th} = 0.01, \ x_0 = 0.05, \pi\tau^2 \simeq 0.3$
  - 'flow' dominates when $x_0 \lesssim (\pi^2/4)\tau^2 \simeq 0.2$, i.e. when

$$\theta_0 \gtrsim \frac{2}{\pi^2}\frac{Q_s}{\bar{\alpha}_s^2\omega_c} \sim \frac{2Q_s}{\omega_c} \simeq 0.2$$

  - 'flow' is built with quanta having $x \sim x_{\rm th}$, or $\omega \sim T \sim 1$ GeV

- Qualitative and even quantitative agreement with the LHC data

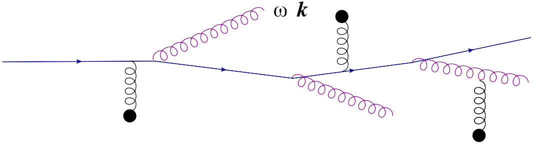




- Such an agreement would be impossible without the 'turbulent flow'

Radiative momentum broadening

- So far: the effects of transverse momentum broadening on the medium–induced radiation
- The opposite effect exists as well: medium–induced gluon emissions contribute to momentum broadening, via their recoil

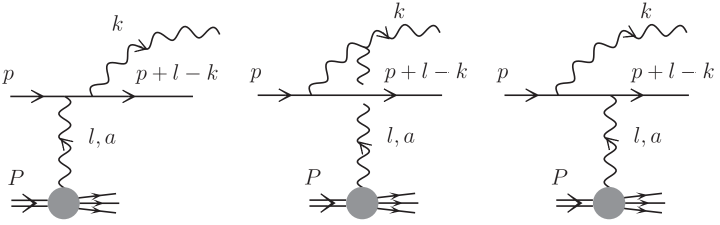


$$\langle p_\perp^2 \rangle_{\rm rad} \sim \int_\omega \int_{\boldsymbol{k}} \boldsymbol{k}^2 \frac{{\rm d}N}{{\rm d}\omega {\rm d}^2\boldsymbol{k}}$$

- A large radiative correction to $\hat{q}$ *(Liou, Mueller, Wu, 13)*
  - formally suppressed by α_s but enhanced by large logarithms coming from integrating over the phase–space
- To evaluate this, we also need the distribution of the radiation in $k_\perp$

The double logarithmic correction

- Dominant effect from relatively hard emissions (large $k_\perp$), as triggered by a single scattering (Gunion–Bertsch spectrum)



$$\omega\frac{\mathrm{d}N}{\mathrm{d}\omega\,\mathrm{d}^2\boldsymbol{k}}\simeq\frac{\alpha_s N_c}{\pi^2}\,\frac{\hat{q}L}{k_\perp^4}\qquad\text{(N.B. : linear in }\hat{q}\text{)}$$

- The radiative contribution to the $p_\perp$–broadening of the quark:

$$\langle p_\perp^2\rangle_{\rm rad}\,=\int_{\omega,\boldsymbol{k}}\boldsymbol{k}^2\,\frac{\mathrm{d}N}{\mathrm{d}\omega\,\mathrm{d}^2\boldsymbol{k}}\sim L\,\alpha_s\hat{q}\int\frac{\mathrm{d}\omega}{\omega}\int\frac{\mathrm{d}^2k_\perp}{k_\perp^2}\equiv L\,\Delta\hat{q}$$

A renormalization group equation for $\hat{q}$

- The limits of the phase–space are simpler in terms of the fluctuation lifetime $\tau = 2\omega/k_\perp^2$ *(below, $\lambda \equiv 1/T$: thermal wavelength)*

$$\frac{\Delta\hat{q}}{\hat{q}} \sim \alpha_s \int_\lambda^L \frac{\mathrm{d}\tau}{\tau} \int_{\hat{q}\tau}^{\hat{q}L} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} = \frac{\alpha_s N_c}{2\pi} \ln^2(LT) \simeq 1$$

  - a relatively large correction $\Longrightarrow$ needs for resummation

- To double–logarithmic accuracy the higher–loop corrections are strongly ordered in lifetimes and transverse momenta

$$\hat{q}(L) = \hat{q}^{(0)} + \bar{\alpha} \int_\lambda^L \frac{\mathrm{d}\tau}{\tau} \int_{\hat{q}\tau}^{\hat{q}L} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \hat{q}_\tau(k_\perp^2)$$

(E.I., arXiv:1403.1996; Blaizot and Mehtar-Tani, arXiv:1403.2323)

- (For experts:) Not the standard DLA eq. (different integration limits)

From fixed to running coupling

- The solution for a fixed QCD coupling α_s *(Liou, Mueller, Wu, 13)*

$$\hat{q}(L) \;=\; \hat{q}^{(0)}\,\frac{1}{\sqrt{\bar\alpha}\,\ln\left(L/\lambda\right)}\,\mathrm{I}_1\left(2\sqrt{\bar\alpha}\,\ln\frac{L}{\lambda}\right) \,\propto\; L^{2\sqrt{\bar\alpha}}$$

  - large anomalous dimension $\gamma_s = 2\sqrt{\bar\alpha} \sim 1$ (with $\bar\alpha = \alpha_s N_c/\pi$)

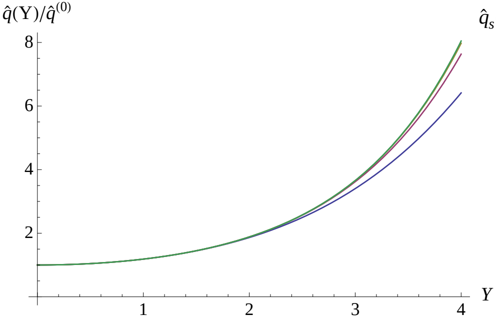
- However, this is qualitatively modified by the running of the coupling *(E.I., D.N. Triantafyllopoulos, to appear)*

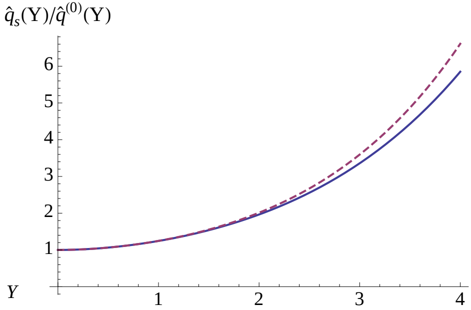
$$\hat{q}(L) \;=\; \hat{q}^{(0)} + \int_\lambda^L \frac{\mathrm{d}\tau}{\tau}\int_{\hat{q}\tau}^{\hat{q}L}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\bar\alpha(k_\perp^2)\,\hat{q}_\tau(k_\perp^2)$$

$$\Longrightarrow\;\ln\hat{q}(L) \,\simeq\, 4\sqrt{b_0\ln\frac{L}{\lambda}} \qquad \text{(slower than any exponential)}$$

- The pre–asymptotic corrections too are under control

From fixed to running coupling





Fixed coupling $\bar{\alpha} = 0.35$

Running coupling (1 loop)

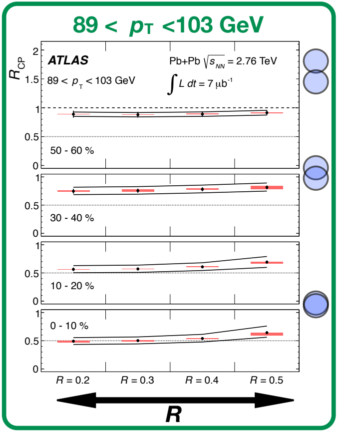
- $Y \equiv \ln(L/\lambda)$: evolution 'time'; at RHIC and LHC, $Y = 2 \div 4$.
- Similar predictions with FC and RC for the physical value $Y = 3$: enhancement by a factor of 3

Conclusions

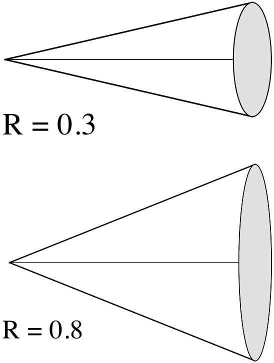
- Remarkable progress in understanding medium–induced jet evolution
  - a new kind of branching process in pQCD
  - hard emissions at small angles (energy loss by leading particle, R_{AA})
  - soft, quasi–democratic, branchings leading to turbulent flow (di-jet asymmetry)
  - probabilistic picture, well suited for Monte Carlo implementations
  - fully 3+1–dim simulations possible $\Longrightarrow$ jet shapes
  - large radiative corrections to $\hat{q}$ which are under control in pQCD
- Many open problems:
  - proper interplay with 'vacuum' radiation
  - fully (3+1)–dim simulations, extensive phenomenology ...

THANK YOU !

- Just a little fraction of the ‘missing energy’ is recovered when gradually increasing the jet opening : most of the energy is lost at large angles

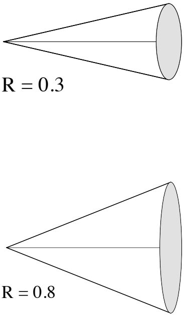


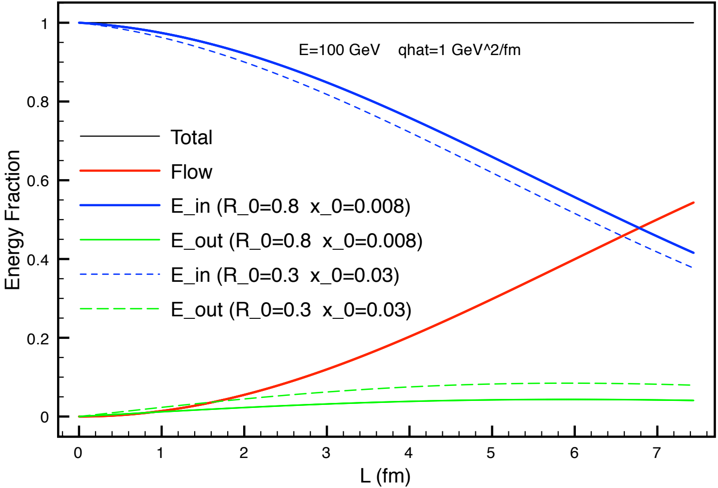
ATLAS, arXiv:1208.1967



- What is the mechanism for energy transport at large angles ?

Energy flow at large angles





- The energy inside the jet is only weakly increasing with the jet angular opening R, within a wide range of values for R ☺