Jet evolution in a dense QCD medium: I

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ALICE Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV (0 - 5%)
STAR Au-Au $\sqrt{s_{NN}} = 200$ GeV (0 - 5%)
PHENIX Au-Au $\sqrt{s_{NN}} = 200$ GeV (0 - 10%)
Pb+Pb collisions at the LHC: \( \sim 20,000 \) hadrons in the detectors

Produced via parton fragmentation & hadronisation

At early stages, all such partons were confined in a small region in space–time \( \rightarrow \) hot and dense partonic matter
Partonic matter in a Heavy Ion Collision

Prior to the collision: 2 Lorentz–contracted nuclei (‘pancakes’)
- ‘Color Glass Condensate’ : highly coherent form of gluonic matter

Right after the collision: non–equilibrium partonic matter
- ‘Glasma’ : color fields break into partons

At later stages ($\Delta t \gtrsim 1$ fm/c) : local thermal equilibrium
- ‘Quark–Gluon Plasma’ (QGP)

Final stage ($\Delta t \gtrsim 10$ fm/c) : hadrons
- ‘final event’, or ‘particle production’
Partonic matter in a Heavy Ion Collision

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How to study these ephemeral partonic stages?
Hard probes

- A space–time picture of a heavy ion collision

Hard partons, photons, leptons created at early times: $\tau \lesssim 1 \text{ fm/c}$

Interact with the surrounding medium on their way to the detectors
Jet quenching

- Hard partons are typically created in pairs which propagate back-to-back in the transverse plane.

- ‘Jet’: ‘leading particle’ + ‘products of fragmentation’

- AA collisions: jet propagation and fragmentation can be modified by the surrounding medium: ‘jet quenching’
Di–hadron azimuthal correlations

- Distribution of pairs of particles w.r.t. the relative angle $\Delta \Phi$ in the transverse plane

- Di–hadron azimuthal correlations at RHIC:
  - $p+p$ or $d+Au$: a peak at $\Delta \Phi = \pi$ ($p_1 + p_2 \simeq 0$)
Di–hadron azimuthal correlations

- Distribution of pairs of particles w.r.t. the relative angle $\Delta \Phi$ in the transverse plane

Di–hadron azimuthal correlations at RHIC:

- Au+Au : the away jet has disappeared!
- Collisions in the medium lead to transverse momentum broadening
Nuclear modification factor

- Use p+p collisions as a benchmark for particle production

$$R_{A+A} \equiv \frac{1}{A^2} \frac{dN_{A+A}/d^2p_\perp d\eta}{dN_{p+p}/d^2p_\perp d\eta}$$

- No suppression for photons, small suppression in peripheral collisions
Nuclear modification factor

- Use p+p collisions as a benchmark for particle production

\[ R_{A+A} \equiv \frac{1}{A^2} \frac{dN_{A+A}/d^2p_\perp d\eta}{dN_{p+p}/d^2p_\perp d\eta} \]

- Strong suppression \((R_{AA} \lesssim 0.2)\) in central collisions
Nuclear modification factor

- Use p+p collisions as a benchmark for particle production

\[
R_{A+A} \equiv \frac{1}{A^2} \frac{dN_{A+A}/d^2p_\perp d\eta}{dN_{p+p}/d^2p_\perp d\eta}
\]

- Large energy loss via interactions in the medium
Hadrons measured with an energy $E$ have been actually produced with a larger energy $E + \epsilon$

$$\frac{d\sigma_{med}(E)}{dE} = \int d\epsilon \mathcal{P}(\epsilon) \frac{d\sigma_{vac}(E + \epsilon)}{dE}$$

$$\frac{d\sigma_{vac}(E)}{dE} \sim \frac{1}{E^n}, \quad n = 7 \div 10$$

- $\mathcal{P}(\epsilon)$: probability density for losing an energy $\epsilon$
- Large $n$ favors small $\epsilon \implies$ one typically measures the leading particle
Central Pb+Pb: ‘mono–jet’ events

The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV
Additional energy imbalance as compared to $p+p$: 20 to 30 GeV

Detailed studies show that the ‘missing energy’ is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

▷ a surprising fragmentation pattern from the standard viewpoint of pQCD
Can one understand such phenomena from first principles?

In **perturbative QCD**, they all find a common denominator: incoherent multiple scattering off the medium constituents.

- random kicks provide **transverse momentum broadening**
- medium induced radiation leading to **large energy loss**
- large emission angles, especially for the **softest emitted quanta**
- color decoherence leading to enhanced jet fragmentation
Assumes that the **couplings** are weak for all elementary processes

- scattering in the medium, emission vertices
Perturbative QCD for jet quenching

- Assumes that the **couplings** are weak for all elementary processes
- Justified (*by asymptotic freedom*) if the jet is **sufficiently energetic** and if the medium is **sufficiently dense**
  - N.B. not clear that this medium is weakly coupled for all purposes
  - the physics of jet quenching is biased towards hard momentum transfers
Assumes that the couplings are weak for all elementary processes

Justified (by asymptotic freedom) if the jet is sufficiently energetic and if the medium is sufficiently dense

Even if the coupling is weak, the pQCD treatment remains elaborated

- no naive perturbative expansion in powers of $\alpha_s = g^2/4\pi$
- high density effects (screening, multiple scattering, ...) and jet evolution (soft multiple emissions, large radiative corrections) must be resummed to all orders in $\alpha_s$
Assumes that the **couplings** are weak for all elementary processes

Justified *(by asymptotic freedom)* if the jet is **sufficiently energetic** and if the medium is **sufficiently dense**

Even if the coupling is weak, the pQCD treatment remains elaborated

Reorganizations of the perturbation theory which might be ambiguous and are certainly complicated ➞ **several approaches in the literature**

- here: mostly the ‘BDMPSZ approach’ to medium-induced gluon radiation and its subsequent developments by many authors
Assumes that the **couplings** are weak for all elementary processes

Justified (by asymptotic freedom) if the jet is **sufficiently energetic** and if the medium is **sufficiently dense**

Even if the coupling is weak, the pQCD treatment remains elaborated

Reorganizations of the perturbation theory which might be ambiguous and are certainly complicated → **several approaches in the literature**

Important simplifications due to the **high energy kinematics**

- eikonal approximation, ‘frozen’ correlations, instantaneous exchanges ...
Assumes that the **couplings** are weak for all elementary processes

Justified *(by asymptotic freedom)* if the jet is **sufficiently energetic** and if the medium is **sufficiently dense**

Even if the coupling is weak, the pQCD treatment remains elaborated

Reorganizations of the perturbation theory which might be ambiguous and are certainly complicated \(\Rightarrow\) several approaches in the literature

Important simplifications due to the **high energy kinematics**

Additional, simplifying, assumptions about the **nature of the medium**

- quark–gluon plasma in thermal equilibrium
- can be relaxed for more realistic, phenomenological, studies
Assumes that the **couplings** are weak for all elementary processes

Justified *(by asymptotic freedom)* if the jet is **sufficiently energetic** and if the medium is **sufficiently dense**

Even if the coupling is weak, the pQCD treatment remains elaborated

Reorganizations of the perturbation theory which might be ambiguous and are certainly complicated \(\Rightarrow\) several approaches in the literature

Important simplifications due to the **high energy kinematics**

Additional, simplifying, assumptions about the **nature of the medium**

So far, mostly a **leading–order formalism** (including resummations), but some **next–to–leading order corrections** are known as well, concerning the medium, the jets, and their mutual interactions

- see also the lectures by Zhong-bo Kang and Jacopo Ghiglieri
L1: Transverse momentum broadening

- most calculations will be explicit on the slides

L2: Medium–induced gluon radiation

- BDMPSZ mechanism
- heuristic discussion: physical considerations, parametric estimates

L3: Jet evolution via multiple branchings

- some recent developments (again, heuristically)
- color decoherence, wave turbulence, relation to di-jet asymmetry

L3 (cont.): NLO corrections to the jet quenching parameter
An energetic quark acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance $L$.

Weakly coupled medium $\Rightarrow$ quasi independent scattering centers

- successive collisions give random kicks
- Brownian motion in $p_\perp$: $\langle p_{\perp}^2 \rangle \simeq \hat{q} \Delta t$

$\hat{q}$: the ‘jet quenching parameter’ (a medium transport coefficient)

- a fundamental quantity for what follows
An energetic quark acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance $L$.

A simple estimate: kinetic theory

- parton mean free path $\ell \sim 1/n\sigma$
- $n$: density of medium constituents; $\sigma$: elastic cross-section
- average (momentum)$^2$ transfer per scattering $\mu^2$

$$\hat{q} \sim \frac{\mu^2}{\ell} = n\sigma\mu^2$$
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- parton mean free path $\ell \sim 1/n\sigma$
- $n$: density of medium constituents; $\sigma$: elastic cross-section
- average (momentum)$^2$ transfer per scattering $\mu^2$

$$\hat{q} \sim \frac{\mu^2}{\ell} = n\sigma\mu^2 = n \int d^2k \frac{d\sigma_{el}}{d^2k}$$
How to study the propagation of an energetic ‘probe’ (quark, gluon, jet) through a dense QCD medium?

- the medium can be a quark–gluon plasma with temperature $T$, but the energetic probe is not a part of the thermal distribution!
- it has an (initial) energy $E \gg T$

**Difficulty**: multiple scattering off the medium constituents

- resummation of the perturbative series to all orders
How to study the propagation of an energetic ‘probe’ (quark, gluon, jet) through a dense QCD medium?

- the medium can be a quark–gluon plasma with temperature $T$, but the energetic probe is not a part of the thermal distribution!
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Main simplification at high–energy: eikonal approximation

- the probe transverse coordinate is not modified by the interactions
A parenthesis on kinematics: **Light–cone variables**

- For relativistic particles \(|v_z| \simeq 1\), it is useful to use LC variables

\[
x^{\mu} = (x^+, x^-, x_\perp)
\]

\[
x^{\pm} = \frac{1}{\sqrt{2}}(t \pm z)
\]

\[
p^{\pm} = \frac{1}{\sqrt{2}}(E \pm p_z)
\]

**dot product:** \(x \cdot p = x^+ p^- + x^- p^+ - x_\perp \cdot p\)

- Our hard probe: a rapid right–mover with \(E \simeq p_z \gg p_\perp (m \simeq 0)\)
  - \(z \simeq t \implies x^- \simeq 0\) (Lorentz contraction) & \(x^+ \simeq \sqrt{2}t\) (LC time)
  - mass–shell condition: \(p^2 = 2p^+ p^- - p_\perp^2 = 0\)

\[
p^+ \simeq \sqrt{2}E \gg p_\perp \gg p^- = \frac{p^2_\perp}{2p^+}
\]
The energetic probe (say, a quark) has a color current which couples to the color field generated by the constituents of the medium.

\[ \mathcal{L}_{\text{int}}(x) = j_\alpha^\mu(x) A_\mu^\alpha(x), \quad j_\alpha^\mu(x) = g \bar{\psi}(x) \gamma^\mu t^\alpha \psi(x) \]

The quark evolution operator in the interaction representation:

\[ e^{-i \hat{H}t} = e^{-i \hat{H}_0 t} \hat{S}(t), \quad \hat{S}(t) = T \int_{-\infty}^{t} dt' \int d^3x \mathcal{L}_{\text{int}}(t', x) \]

High–energy formalism: replace \( t \to x^+ \) and \( d^3x \to dx^- d^2x \)
Eikonal approximation

- The individual collisions are relatively soft: \( k_\perp \sim m_D \ll E \)

- a straightline trajectory with \( v^\mu = \delta^\mu_+ \), \( x^- = 0 \), \( x = x_0 \)

\[
\hat{j}_a^\mu(x) \simeq \delta^\mu_+ g t^a \delta(x^-) \delta^{(2)}(x - x_0) \in \text{su}(N_c)
\]

- The \( S \)-matrix reduces to a Wilson line in the fundamental repres.

\[
\hat{S}(x^+) \simeq T \exp \left\{ i g \int_{-\infty}^{x^+} dz^+ A^-_a(z^+, x_0) t^a \right\} \equiv V(x^+, x_0)[A^-]
\]

- Time–ordered exponential, all orders in \( A^- \) (multiple scattering)
Eikonal approximation

- The individual collisions are relatively soft: \( k_\perp \sim m_D \ll E \)

  ▶ a straightline trajectory with \( v^\mu = \delta^{\mu+} \), \( x^- = 0 \), \( x = x_0 \)

\[
j_a^\mu(x) \simeq \delta^{\mu+} g t^a \delta(x^-) \delta^{(2)}(x - x_0) \in \text{su}(N_c)
\]

- Best understood with a discretization of time: \( x_n^+ = n\epsilon, n = 0, 1, \cdots N \)

\[
V_N = e^{ig\epsilon A_N^-} e^{ig\epsilon A_{N-1}^-} \cdots e^{ig\epsilon A_1^-} e^{ig\epsilon A_0^-} \quad (A_n^- \equiv A_a(x_n^+) t^a)
\]

▶ a sequence of infinitesimal color rotations
More on the Wilson lines

- **Elastic scattering**: the $S$–matrix is a pure phase
  - color rotation of the quark wavefunction

  \[ \psi_i(x^+; x_0) = V_{ij}(x^+, x_0) \psi_j(0; x_0) \]

- **Physics**: precession of the quark color current

  \[ j^+_a(x^+) = U_{ab}(x^+) j^+_b(-\infty) \implies (\partial^- - igA^-)_{ab} j^+_b(x^+) = 0 \]

  [Hint : use $V^\dagger t^a V = U_{ab} t^b$ with $U$ the Wilson line in the adjoint repres.]

- ... as required by covariant current conservation: $D_\mu j^\mu = 0$

- The fields $A^-_a$ are **randomly distributed** (since so are their sources)
  - say, according to the thermal distribution in the case of a QGP

- Cross–sections are obtained after **averaging over the background field**
Transverse momentum broadening

- Direct amplitude (DA) \( \times \) Complex conjugate amplitude (CCA) :

\[
\mathbf{p}_\perp = 0 \quad \mathbf{p}_\perp = \mathbf{0} \quad L = 0
\]

- The \( p_\perp \)-spectrum of the quark after crossing the medium:

\[
\frac{dN}{d^2 p} = \frac{1}{(2\pi)^2} \int_\mathcal{R} e^{-ip \cdot \mathbf{r}} \langle S_{xy} \rangle, \quad S_{xy} \equiv \frac{1}{N_c} \text{tr}(V_x V_y^\dagger)
\]

- sum over the final color indices, average over the initial ones
- average over the distribution of the medium field \( A_a^- \)
Transverse momentum broadening

- Direct amplitude (DA) \times Complex conjugate amplitude (CCA):

\[
p_{\perp} = 0 \\
x_{\perp} = 0 \quad L \quad \infty \quad L \quad 0
\]

- The \( p_{\perp} \)-spectrum of the quark after crossing the medium:

\[
\frac{dN}{d^2p} = \frac{1}{(2\pi)^2} \int_{r} e^{-ip \cdot r} \langle S_{xy} \rangle, \quad S_{xy} \equiv \frac{1}{N_c} \text{tr}(V_x V_y^\dagger)
\]

▷ check normalization: \( \int_{p}(dN/d^2p) = 1 \) since \( S_{xy} = 1 \) when \( x = y \)
Formally, $\langle S_{xy} \rangle$ is the average $S$–matrix for a $q\bar{q}$ color dipole

- ‘the quark at $x$’ : the physical quark in the DA
- ‘the antiquark at $y$’ : the physical quark in the CCA

Quark cross–section $\longleftrightarrow$ dipole amplitude : a useful analogy
With due respect to the medium

- A collection of quasi–independent color charges (quarks and gluons)
  - e.g. a nearly ideal quark–gluon plasma
- Even at weak coupling, some effects of the interactions are essential
  - collective phenomena leading to the screening of the gauge interactions

\[ A^0(r) = \int d^3r' \frac{e^{-m_D|r-r'|}}{|r - r'|} \rho(r') = \int \frac{d^3k}{(2\pi)^3} e^{i k \cdot r} \frac{\rho(k)}{k^2 + m_D^2} \]

- Weakly coupled QGP: \( m_D \sim gT \) (see lectures by Ghiglieri)
  - Debye mass acts as an ‘infrared’ (\( k \to 0 \)) cutoff

\[ V(r) = \exp\left( - \frac{m_{\text{debye}} r}{r} \right) \]
View the scattering process in a boosted Lorentz frame, where the medium is a rapid ‘left mover’: $v_z < 0, \ |v_z| \simeq 1$

The color current density of the medium: $J_a^\mu(x) \simeq \delta^{\mu-} \rho_a(x^+, \mathbf{x}_\perp)$

The gauge field in gauge $A^+ = 0$: $A_a^\mu(x) = \delta^{\mu-} A_a^- (x^+, \mathbf{x}_\perp)$

\[ A_a^- (x^+, \mathbf{x}_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} e^{i k_\perp \cdot \mathbf{x}_\perp} \frac{\rho_a(x^+, \mathbf{k}_\perp)}{k_\perp^2 + m_D^2} \]

▷ local in $x^+$ due to Lorentz–contraction

▷ Coulomb propagator in two (transverse) directions

The 2–point ‘correlation’ function of independent color sources:

\[ \langle \rho_a(x^+, \mathbf{x}_\perp) \rho_b(y^+, \mathbf{y}_\perp) \rangle = g^2 n_0 \delta_{ab} \delta(x^+ - y^+) \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp) \]

▷ $n_0 \sim T^3$: quark and gluon densities weighted with color factors
View the scattering process in a boosted Lorentz frame, where the medium is a rapid ‘left mover’: \( v_z < 0, \quad |v_z| \simeq 1 \)

The color current density of the medium: \( J^\mu_a(x) \simeq \delta^\mu - \rho_a(x^+, \vec{x}_\perp) \)

The gauge field in gauge \( A^+ = 0 \): \( A^\mu_a(x) = \delta^\mu - A_a^-(x^+, \vec{x}_\perp) \)

\[
A_a^-(x^+, \vec{x}_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^2} e^{i k_\perp \cdot \vec{x}_\perp} \frac{\rho_a(x^+, k_\perp)}{k^2_\perp + m^2_D}
\]

\( \triangleright \) local in \( x^+ \) due to Lorentz–contraction

\( \triangleright \) Coulomb propagator in two (transverse) directions

The ensuing 2–point correlation function of the gauge fields:

\[
\langle A_a^-(x^-, x^+, \vec{x}_\perp) A_b^-(x^-, y^+, \vec{y}_\perp) \rangle = g^2 n_0 \delta_{ab} \delta(x^+ - y^+) \gamma(\vec{x}_\perp - \vec{y}_\perp)
\]

\( \gamma(k_\perp) \equiv \frac{1}{(k^2_\perp + m^2_D)^2} : \) Coulomb propagator squared

\( \triangleright \) 2 gluon exchange with a same medium constituent
The dipole $S$–matrix (1)

$$\langle S_{xy} \rangle = \frac{1}{N_c} \left\langle \text{tr}(V(x)V^\dagger(y)) \right\rangle, \quad V(x) = T e^{ig \int dx^+ A_a^-(x^+,x)t^a}$$

- Correlations are local in $x^+ \implies$ discretize the respective range:

  $$L = N\epsilon, \ x_n^+ = n\epsilon, \ A_n^-(x) \equiv A_a^-(x_n^+,x)t^a$$

$$V_N(x) = e^{ig\epsilon A_N^-(x)} e^{ig\epsilon A_{N-1}^-(x)} \cdots e^{ig\epsilon A_0^-(x)} = e^{ig\epsilon A_N^-(x)} V_{N-1}(x)$$

$$S_N(x, y) = \frac{1}{N_c} \text{tr} \left( e^{ig\epsilon A_N^-(x)} S_{N-1}(x, y) e^{-ig\epsilon A_N^-(y)} \right)$$

- The Gaussian correlations in such discretized notations:

  $$\langle A_{a,n}^-(x) A_{b,m}^-(y) \rangle = g^2 n_0 \delta_{ab} \frac{1}{\epsilon} \delta_{nm} \gamma(x - y)$$

- Correlations at different times factorize from each other !
The dipole $S$–matrix (2)

- Recurrence formula for $\langle S_n(x, y) \rangle$:

$$\langle S_n(x, y) \rangle = \frac{1}{N_c} \left\langle \text{tr} \left[ \text{e}^{ig\epsilon A_n^-(x)} \text{e}^{-ig\epsilon A_n^-(y)} \right] \right\rangle \langle S_{n-1}(x, y) \rangle$$

- Expand up to quadratic order in $\epsilon A_n^-$, i.e. to linear order in $\epsilon$
  - the linear terms from the 2 exponentials average with each other

$$\frac{1}{N_c} \left\langle \text{tr}(ig\epsilon A_{n,a}^-(x)t^a) (-ig\epsilon A_{n,b}^-(y)t^b) \right\rangle = C_F(g^2\epsilon)(g^2n_0)\gamma(x - y)$$

  - the quadratic terms self–average (‘tadpoles’)

$$\frac{(ig\epsilon)^2}{2} \frac{1}{N_c} \left\langle \text{tr}(A_{n,a}^-(x)t^a A_{n,b}^-(x)t^b) \right\rangle = -C_F \frac{g^2\epsilon}{2} (g^2n_0)\gamma(0)$$

- Evolution equation for $\langle S_n(x, y) \rangle$ w.r.t $x^+$:

$$\frac{\langle S_n(x, y) \rangle - \langle S_{n-1}(x, y) \rangle}{\epsilon} = -g^4C_Fn_0[\gamma(0) - \gamma(x - y)] \langle S_{n-1}(x, y) \rangle$$
The dipole $S$–matrix (3)

- Continuum limit $\epsilon \to 0 \implies$ equation for $\langle S_t(x, y) \rangle \ (t \equiv x^+ \in [0, L])$

\[
\frac{\partial \langle S_t(x, y) \rangle}{\partial t} = -g^4 C_F n_0 \left[ \gamma(0) - \gamma(x - y) \right] \langle S_t(x, y) \rangle
\]

- The solution $\langle S(x, y) \rangle \equiv \langle S_L(x, y) \rangle$ is clearly an exponential:

\[
\langle S(x, y) \rangle = \exp \left\{ -g^4 C_F n_0 L \left[ \gamma(0) - \gamma(x - y) \right] \right\}
\]

- $\gamma(0) > \gamma(x - y) \implies$ the exponent is positive $\implies$ attenuation
- $\langle S(x, y) \rangle$ is a function of the dipole size $r = x - y$ (by homogeneity)

- Exponent: amplitude for a single scattering via 2–gluon exchange
  - the dipole is a color–singlet and must remains so after each scattering
  $\implies$ a single scattering involves the exchange of two gluons

- Independent successive scatterings exponentiate (Glauber series)
Diagrammatic interpretation

- The amplitude for a single scattering:

\[ \langle T(x, y) \rangle_0 = g^4 C_F n_0 L [\gamma(0) - \gamma(x - y)] \]

- The dipole is a color–singlet and must remain so after the scattering

  \( \implies \) a single scattering involves the exchange of two gluons

- The medium correlations are Gaussian

  \( \implies \) both gluons are exchanged with a same ‘color source’ from the medium
Diagrammatic interpretation

- The amplitude for a single scattering:

\[
\langle T(x, y) \rangle_0 = g^4 C_F n_0 L [\gamma(0) - \gamma(x - y)]
\]

- The dipole is a color–singlet and must remain so after the scattering

\[\implies \text{a single scattering involves the exchange of two gluons}\]

- The medium correlations are local in time \((x^+)\)

\[\implies \text{instantaneous exchange between the quark and the antiquark}\]
Diagrammatic interpretation

- The amplitude for a single scattering:

\[ \langle T(x, y) \rangle_0 = g^4 C_F n_0 L [\gamma(0) - \gamma(x - y)] \]

- The dipole is a color-singlet and must remain so after the scattering

\[ \implies \text{a single scattering involves the exchange of two gluons} \]

- The two gluons can also be exchanged with a same fermion (\( q \) or \( \bar{q} \))
Diagrammatic interpretation

- The amplitude for a single scattering:

\[
\langle T(x, y) \rangle_0 = g^4 C_F n_0 L [\gamma(0) - \gamma(x - y)]
\]

- The dipole is a color–singlet and must remain so after the scattering.

\[\implies \text{a single scattering involves the exchange of two gluons}\]

- Effectively, a self–energy tadpole: \(\gamma(0)\)
The jet quenching parameter \(1\)

\[
\langle S(r) \rangle = \exp \left\{ -g^4 C_F n_0 L \int \frac{d^2 k}{(2\pi)^2} \frac{1}{(k^2 + m_D^2)^2} \left( 1 - e^{ik \cdot r} \right) \right\}
\]

- We are interested in relatively small dipole sizes \(r \sim 1/p_\perp \ll 1/m_D\)
  - \(p_\perp \gg m_D\) : the momentum accumulated after many scatterings, each one contributing an amount \(\sim m_D\)

- The integral is dominated by \(k_\perp\) in the range \(m_D \ll k_\perp \ll 1/r\)
  - this range produces a large logarithm \(\ln(1/rm_D) \gg 1\)
  - it dominates to leading logarithmic accuracy: up to terms of \(\mathcal{O}(1)\)

- Within this range one can expand the exponential to quadratic order:
  \[
  1 - e^{ik \cdot r} \simeq -ik \cdot r + \frac{1}{2} (k \cdot r)^2 \longrightarrow \frac{1}{4} k_\perp^2 r^2
  \]
The jet quenching parameter (2)

- To leading logarithmic accuracy, the dipole $S$–matrix reads

$$\langle S(r) \rangle \simeq \exp \left\{ -\frac{1}{4} L \hat{q}(1/r^2) r^2 \right\}$$

- The jet quenching parameter $\hat{q}$:

$$\hat{q}(1/r^2) \equiv g^4 C_F n_0 \int^{1/r^2} \frac{d^2 k}{(2\pi)^2} \frac{k^2}{(k^2 + m_D^2)^2} \simeq 4\pi \alpha_s^2 C_F n_0 \ln \frac{1}{r^2 m_D^2}$$

  △ a property of the medium $(m_D, n_0)$ measured on the resolution scale $r$

- The typical value of $r$ is fixed by multiple scattering

$$\frac{dN}{d^2 p} = \frac{1}{(2\pi)^2} \int e^{-i\mathbf{p} \cdot \mathbf{r}} e^{-\frac{1}{4} L \hat{q}(1/r^2) r^2}$$

  △ dominated by values $r \sim 1/Q_s$ such that the exponent is of $O(1)$
The saturation momentum

\[ Q_s^2 = L \hat{q}(Q_s^2) = 4\pi \alpha_s^2 C_F n_0 L \ln \frac{Q_s^2}{m_D^2} \propto L \ln L \]

- For the dipole: the borderline of multiple scattering
- For the quark: the typical transverse momentum broadening

\[
\frac{dN}{d^2p} \approx \frac{1}{(2\pi)^2} \int e^{-ip \cdot r} e^{-\frac{1}{4}Q_s^2 r^2} = \frac{1}{\pi Q_s^2} e^{-p^2/Q_s^2}
\]

- Gaussian \[\rightarrow\] a random walk in \(p_\perp\): \[\langle p_\perp^2 \rangle = Q_s^2 = \hat{q}(Q_s^2)L\]

- The physical jet quenching parameter: \(\hat{q}(Q_s^2) \propto \ln L\)
  \[\rightarrow\] weak dependence upon the medium size \(L\)

- The physics of \(p_\perp\)-broadening is mildly non-local in time
  \[\rightarrow\] it ‘knows’ about the overall size of the medium
So far, we implicitly assumed that $p_\perp$ is not much larger than $Q_s$

- the typical situation: $p_\perp$ gets accumulated via rescattering in the medium

When $p_\perp \gg Q_s$, the Fourier transform is cut off by the complex exponential $e^{ip\cdot r}$ already for small dipole sizes $r_\perp \ll 1/Q_s$

The $S$–matrix can be evaluated in the single scattering approximation:

$$\langle S(r) \rangle \simeq 1 - \frac{1}{4} L \hat{q}(1/r^2) r^2$$

$$\frac{dN}{d^2p} \simeq \frac{\alpha_s^2 C_F}{4\pi} n_0 L \int_r e^{-ip\cdot r} (-r^2) \ln \frac{1}{r^2m_D^2} = \frac{4\alpha_s^2 C_F n_0 L}{p^4}$$
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- N.B. the logarithmic dependence of $\hat{q}(1/r^2)$ is now essential

The high–$p_\perp$ tail is generated via rare but very hard collisions