Jet evolution in a dense QCD medium: I

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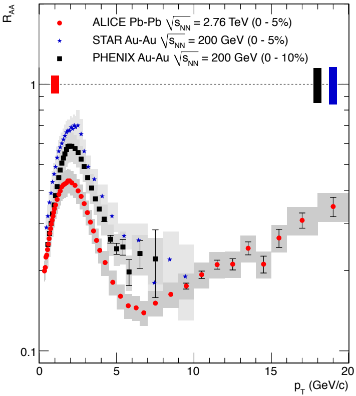
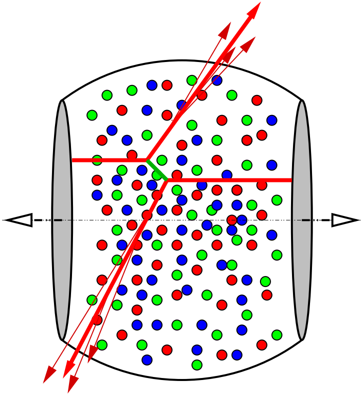
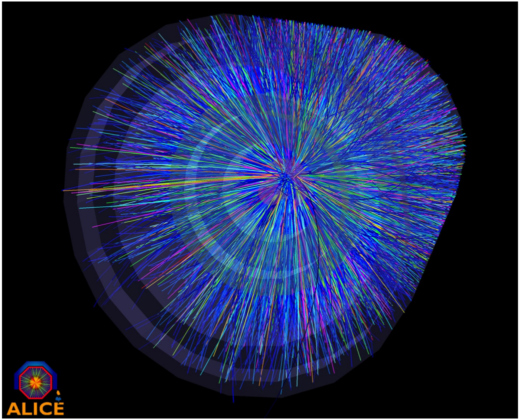


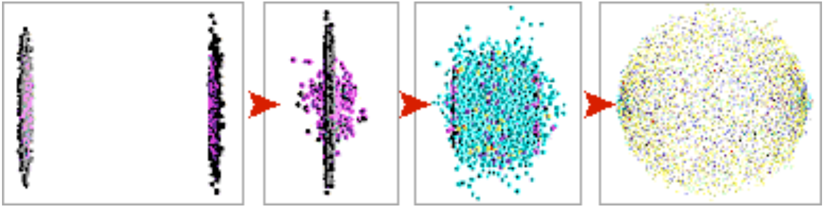


Heavy ion collisions @ the LHC



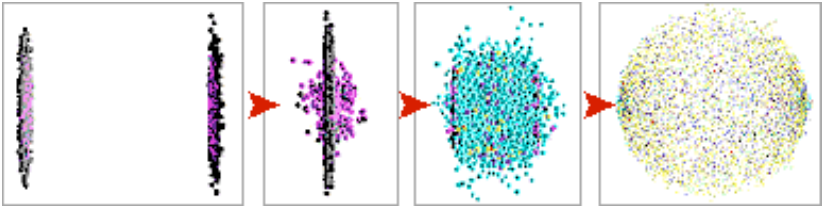
- Pb+Pb collisions at the LHC: $\sim 20,000$ hadrons in the detectors
- Produced via parton fragmentation & hadronisation
- At early stages, all such partons were confined in a small region in space–time $\Longrightarrow$ hot and dense partonic matter

Partonic matter in a Heavy Ion Collision



- Prior to the collision: 2 Lorentz–contracted nuclei ('pancakes')
  - 'Color Glass Condensate' : highly coherent form of gluonic matter
- Right after the collision: non–equilibrium partonic matter
  - 'Glasma' : color fields break into partons
- At later stages ($\Delta t \gtrsim 1$ fm/c) : local thermal equilibrium
  - 'Quark–Gluon Plasma' (QGP)
- Final stage ($\Delta t \gtrsim 10$ fm/c) : hadrons
  - 'final event', or 'particle production'

Partonic matter in a Heavy Ion Collision



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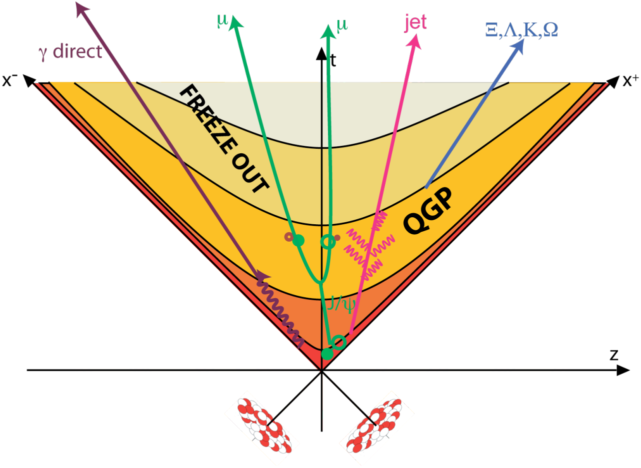
- At later stages ($\Delta t \gtrsim 1$ fm/c) : local thermal equilibrium
  - 'Quark–Gluon Plasma' (QGP)
- How to study these ephemeral partonic stages ?

Hard probes

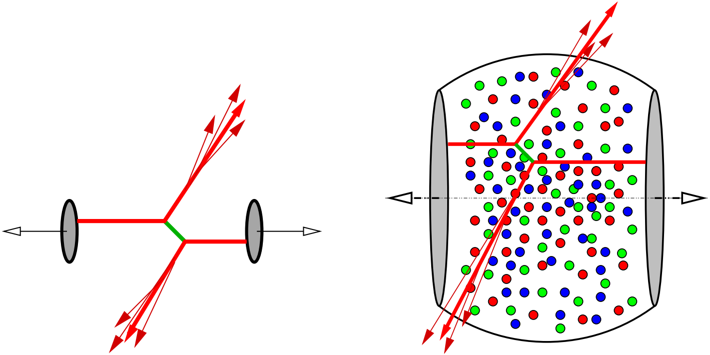
- A space–time picture of a heavy ion collision



- Hard partons, photons, leptons created at early times : $\tau \lesssim 1$ fm/c
- Interact with the surrounding medium on their way to the detectors

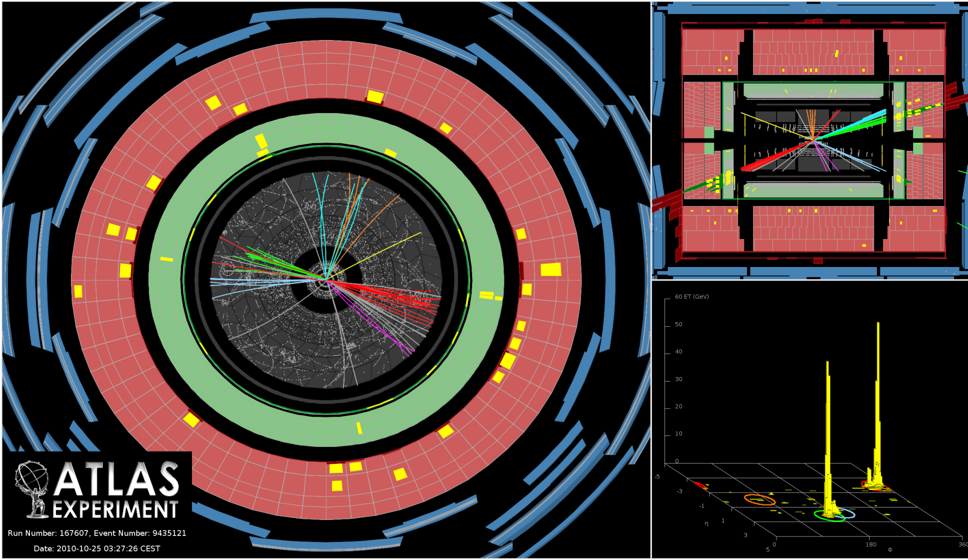
Jet quenching

- Hard partons are typically created in pairs which propagate back–to–back in the transverse plane



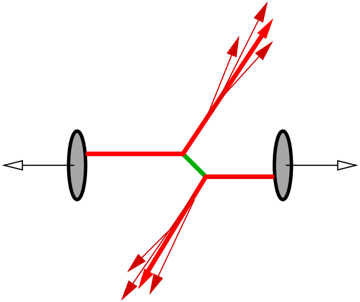
- 'Jet': 'leading particle' + 'products of fragmentation'
- AA collisions : jet propagation and fragmentation can be modified by the surrounding medium: 'jet quenching'

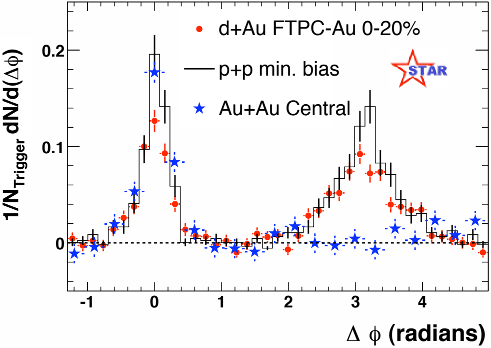
Di–jets in p+p collisions at the LHC



Di–hadron azimuthal correlations

- Distribution of pairs of particles w.r.t. the relative angle $\Delta\Phi$ in the transverse plane

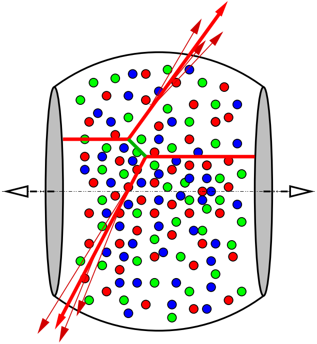


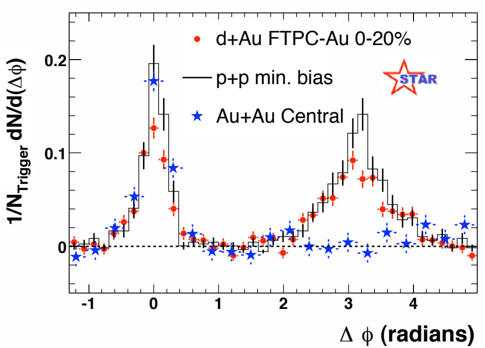


- Di–hadron azimuthal correlations at RHIC:
  - p+p or d+Au : a peak at $\Delta\Phi = \pi$ $\quad (\boldsymbol{p}_1 + \boldsymbol{p}_2 \simeq 0)$

Di–hadron azimuthal correlations

- Distribution of pairs of particles w.r.t. the relative angle $\Delta\Phi$ in the transverse plane



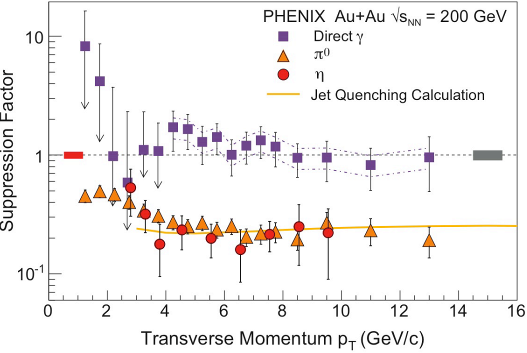


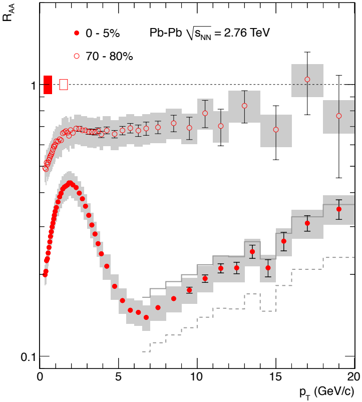
- Di–hadron azimuthal correlations at RHIC:
  - Au+Au : the away jet has disappeared !
- Collisions in the medium lead to transverse momentum broadening

Nuclear modification factor

- Use p+p collisions as a benchmark for particle production

$$R_{\mathrm{A+A}} \equiv \frac{1}{A^2} \frac{\mathrm{d}N_{\mathrm{A+A}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{\mathrm{p+p}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$





- No suppression for photons, small suppression in peripheral collisions

Nuclear modification factor

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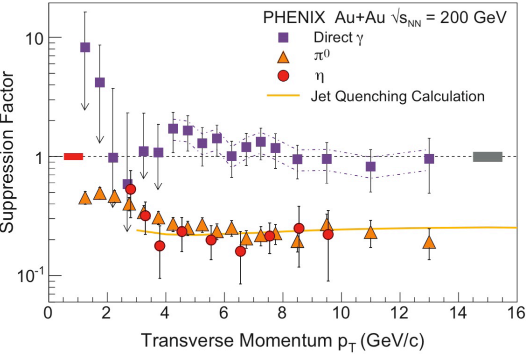

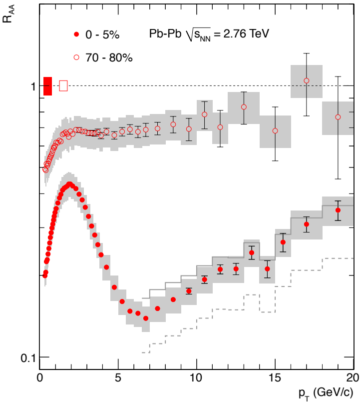

- Strong suppression $(R_{AA} \lesssim 0.2)$ in central collisions

Nuclear modification factor

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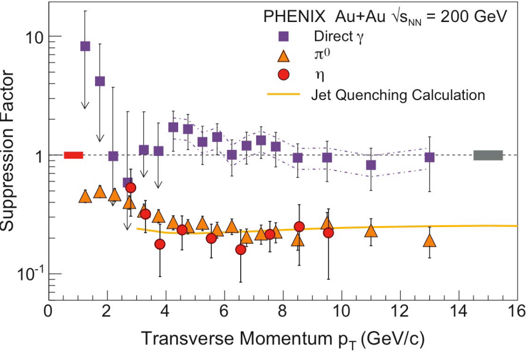
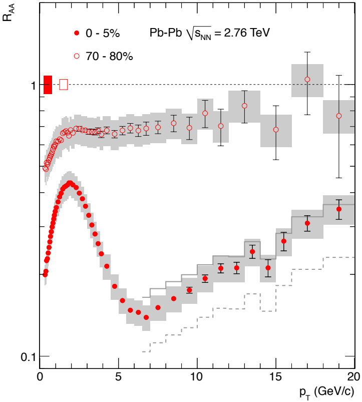
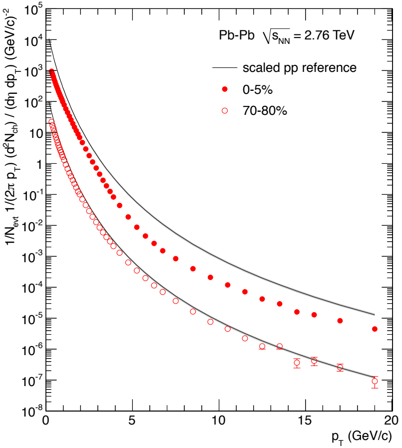




- Large energy loss via interactions in the medium

Energy loss



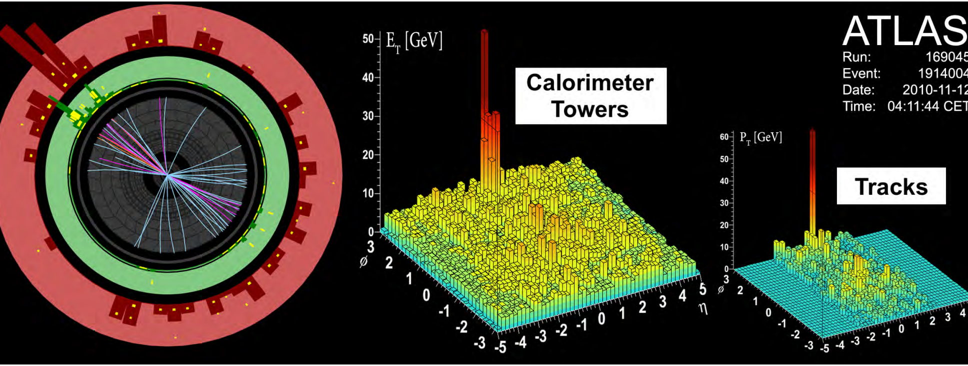
- Hadrons measured with an energy E have been actually produced with a larger energy $E+\epsilon$

$$\frac{\mathrm{d}\sigma^{\mathrm{med}}(E)}{\mathrm{d}E} = \int \mathrm{d}\epsilon\, \mathcal{P}(\epsilon)\, \frac{\mathrm{d}\sigma^{\mathrm{vac}}(E+\epsilon)}{\mathrm{d}E}$$

$$\frac{\mathrm{d}\sigma^{\mathrm{vac}}(E)}{\mathrm{d}E} \sim \frac{1}{E^n}, \qquad n = 7 \div 10$$

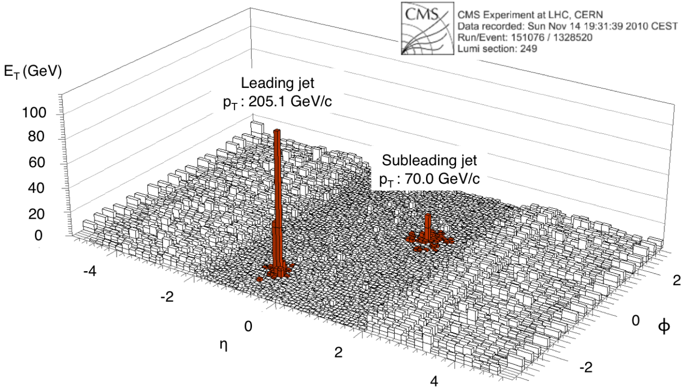
- $\mathcal{P}(\epsilon)$: probability density for losing an energy ϵ
- Large n favors small $\epsilon \Longrightarrow$ one typically measures the leading particle

Di–jet asymmetry *(ATLAS)*



- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry *(CMS)*

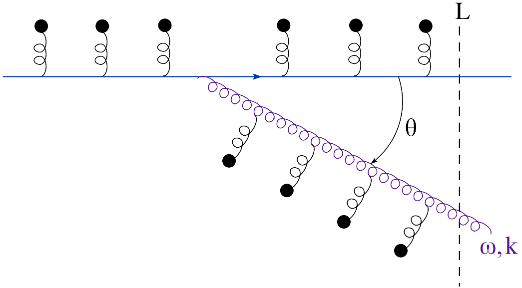


- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

  ▷ a surprising fragmentation pattern from the standard viewpoint of pQCD

Jet quenching in pQCD

- Can one understand such phenomena from first principles ?
- In perturbative QCD, they all find a common denominator: incoherent multiple scattering off the medium constituents



  - random kicks provide transverse momentum broadening
  - medium induced radiation leading to large energy loss
  - large emission angles, especially for the softest emitted quanta
  - color decoherence leading to enhanced jet fragmentation

Perturbative QCD for jet quenching

- Assumes that the couplings are weak for all elementary processes
  - scattering in the medium, emission vertices

Perturbative QCD for jet quenching

- Assumes that the couplings are weak for all elementary processes
- Justified (by asymptotic freedom) if the jet is sufficiently energetic and if the medium is sufficiently dense
  - N.B. not clear that this medium is weakly coupled for all purposes
  - the physics of jet quenching is biased towards hard momentum transfers

Perturbative QCD for jet quenching

- Assumes that the couplings are weak for all elementary processes
- Justified (by asymptotic freedom) if the jet is sufficiently energetic and if the medium is sufficiently dense
- Even if the coupling is weak, the pQCD treatment remains elaborated
  - no naive perturbative expansion in powers of $\alpha_s = g^2/4\pi$
  - high density effects (screening, multiple scattering, ...) and jet evolution (soft multiple emissions, large radiative corrections) must be resummed to all orders in α_s

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- Even if the coupling is weak, the pQCD treatment remains elaborated
- Reorganizations of the perturbation theory which might be ambiguous and are certainly complicated $\Longrightarrow$ several approaches in the literature
  - here : mostly the 'BDMPSZ approach' to medium–induced gluon radiation and its subsequent developments by many authors

Perturbative QCD for jet quenching

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- Important simplifications due to the high energy kinematics
    - eikonal approximation, 'frozen' correlations, instantaneous exchanges ...

Perturbative QCD for jet quenching

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- Important simplifications due to the high energy kinematics
- Additional, simplifying, assumptions about the nature of the medium
    - quark–gluon plasma in thermal equilibrium
    - can be relaxed for more realistic, phenomenological, studies

Perturbative QCD for jet quenching

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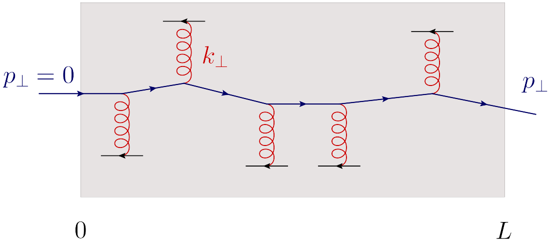
- Important simplifications due to the high energy kinematics
- Additional, simplifying, assumptions about the nature of the medium
- So far, mostly a leading–order formalism (including resummations), but some next–to–leading order corrections are known as well, concerning the medium, the jets, and their mutual interactions
  - see also the lectures by Zhong-bo Kang and Jacopo Ghiglieri

Outline

- L1: Transverse momentum broadening
  - most calculations will be explicit on the slides
- L2: Medium–induced gluon radiation
  - BDMPSZ mechanism
  - heuristic discussion: physical considerations, parametric estimates
- L3: Jet evolution via multiple branchings
  - some recent developments (again, heuristically)
  - color decoherence, wave turbulence, relation to di-jet asymmtery
- L3 (cont.) : NLO corrections to the jet quenching parameter

$p_\perp$–broadening: Introduction (1)

- An energetic quark acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance L



- Weakly coupled medium $\Rightarrow$ quasi independent scattering centers

  ▷ successive collisions give random kicks

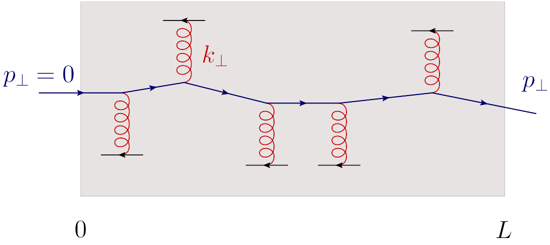
  ▷ Brownian motion in $p_\perp$: $\langle p_\perp^2 \rangle \simeq \hat{q}\,\Delta t$

- $\hat{q}$: the 'jet quenching parameter' (a medium transport coefficient)

  ▷ a fundamental quantity for what follows

$p_\perp$–broadening: Introduction (2)

- An energetic quark acquires a transverse momentum $p_\perp$ via collisions in the medium, after propagating over a distance L



- A simple estimate: kinetic theory
  - parton mean free path $\ell \sim 1/n\sigma$
  - n : density of medium constituents; σ : elastic cross–section
  - average $(\text{momentum})^2$ transfer per scattering μ^2

$$\hat{q} \simeq \frac{\mu^2}{\ell} = n\sigma\mu^2$$

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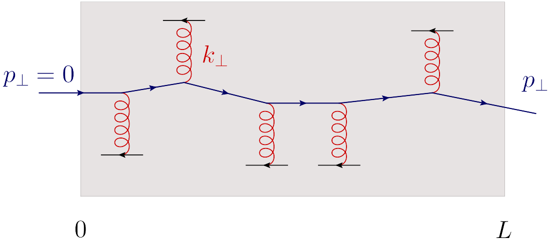

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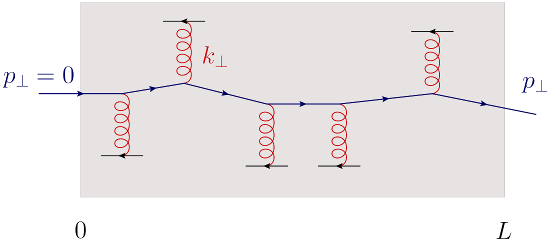
$$\hat{q} \simeq \frac{\mu^2}{\ell} = n\sigma\mu^2 = n\int \mathrm{d}^2\boldsymbol{k}\; \boldsymbol{k}^2\, \frac{\mathrm{d}\sigma_{\rm el}}{\mathrm{d}^2\boldsymbol{k}}$$

$p_\perp$–broadening: Introduction (3)

- How to study the propagation of an energetic 'probe' (quark, gluon, jet) through a dense QCD medium ?

  ▷ the medium can be a quark–gluon plasma with temperature T, but the energetic probe is not a part of the thermal distribution !

  ▷ it has an (initial) energy $E \gg T$



- Difficulty : multiple scattering off the medium constituents

  ▷ resummation of the perturbative series to all orders

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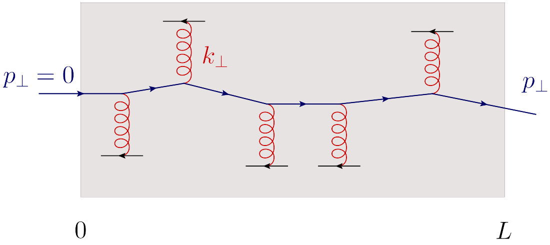
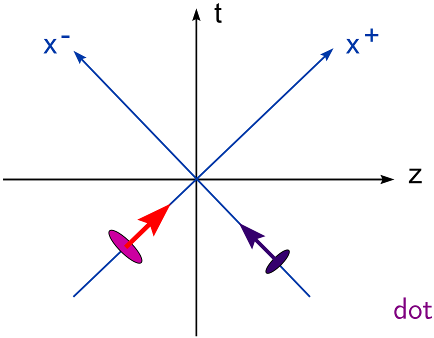


- Main simplification at high–energy : eikonal approximation

  ▷ the probe transverse coordinate is not modified by the interactions

A parenthesis on kinematics: **Light–cone variables**

- For relativistic particles ($|v_z| \simeq 1$), it is useful to use LC variables



$$x^\mu = (x^+, x^-, \boldsymbol{x}_\perp)$$

$$x^\pm = \frac{1}{\sqrt{2}}(t \pm z)$$

$$p^\pm = \frac{1}{\sqrt{2}}(E \pm p_z)$$

dot product: $x \cdot p = x^+ p^- + x^- p^+ - \boldsymbol{x} \cdot \boldsymbol{p}$

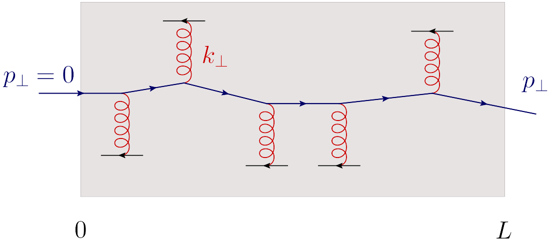
- Our hard probe: a rapid right–mover with $E \simeq p_z \gg p_\perp$ $(m \simeq 0)$
  - $z \simeq t \implies x^- \simeq 0$ (Lorentz contraction) & $x^+ \simeq \sqrt{2}t$ (LC time)
  - mass–shell condition: $p^2 = 2p^+p^- - \boldsymbol{p}^2 = 0$

$$p^+ \simeq \sqrt{2}E \gg p_\perp \gg p^- = \frac{p_\perp^2}{2p^+}$$

The S–matrix

- The energetic probe (say, a quark) has a color current which couples to the color field generated by the constituents of the medium.

$$\mathcal{L}_{\rm int}(x) = j^\mu_a(x) A^a_\mu(x)\,, \qquad j^\mu_a(x) = g\bar\psi(x)\gamma^\mu t^a\psi(x)$$



- The quark evolution operator in the interaction representation :

$$\mathrm{e}^{-\mathrm{i}\hat{H}t} = \mathrm{e}^{-\mathrm{i}\hat{H}_0 t}\,\hat{S}(t)\,, \quad \hat{S}(t) = \mathrm{T}\,\mathrm{e}^{\mathrm{i}\int_{-\infty}^{t}\mathrm{d}t'\int \mathrm{d}^3\mathbf{x}\,\mathcal{L}_{\rm int}(t',\mathbf{x})}$$

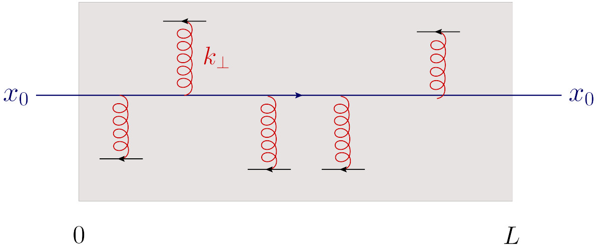
- High–energy formalism: replace $t \to x^+$ and $\mathrm{d}^3\mathbf{x} \to \mathrm{d}x^-\mathrm{d}^2\boldsymbol{x}$

Eikonal approximation

- The individual collisions are relatively soft : $k_\perp \sim m_D \ll E$

  ▷ a straightline trajectory with $v^\mu = \delta^{\mu +}$, $x^- = 0$, $\boldsymbol{x} = \boldsymbol{x}_0$

$$j_a^\mu(x) \simeq \delta^{\mu +} g t^a \delta(x^-) \delta^{(2)}(\boldsymbol{x} - \boldsymbol{x}_0) \in \mathrm{su}(N_c)$$



- The S–matrix reduces to a Wilson line in the fundamental repres.

$$\hat{S}(x^+) \simeq \mathrm{T} \exp \left\{ \mathrm{i}g \int_{-\infty}^{x^+} \mathrm{d}z^+ \, A_a^-(z^+, \boldsymbol{x}_0) t^a \right\} \equiv V(x^+, \boldsymbol{x}_0)[A^-]$$

- Time–ordered exponential, all orders in A^- (multiple scattering)

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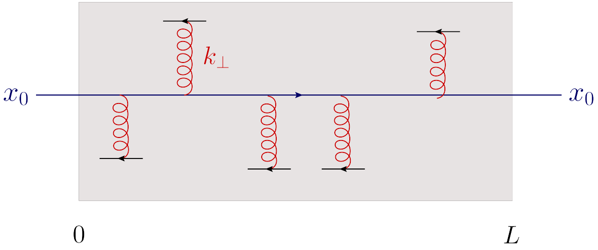


- Best understood with a discretization of time: $x_n^+ = n\epsilon$, $n = 0, 1, \cdots N$

$$V_N = \mathrm{e}^{\mathrm{i}g\epsilon A_N^-}\, \mathrm{e}^{\mathrm{i}g\epsilon A_{N-1}^-} \ldots \mathrm{e}^{\mathrm{i}g\epsilon A_1^-}\, \mathrm{e}^{\mathrm{i}g\epsilon A_0^-} \qquad \left(A_n^- \equiv A_a^-(x_n^+) t^a\right)$$

  ▷ a sequence of infinitesimal color rotations

More on the Wilson lines

- Elastic scattering : the S–matrix is a pure phase

  $\triangleright$ color rotation of the quark wavefunction

$$\psi_i(x^+;\boldsymbol{x}_0) \,=\, V_{ij}(x^+,\boldsymbol{x}_0)\,\psi_j(0;\boldsymbol{x}_0)$$

- Physics: precession of the quark color current

$$j_a^+(x^+) = U_{ab}(x^+)\,j_b^+(-\infty) \;\Longrightarrow\; \left(\partial^- - igA^-\right)_{ab} j_b^+(x^+) = 0$$

  [Hint : use $V^\dagger t^a V = U_{ab} t^b$ with U the Wilson line in the adjoint repres.]

- ... as required by covariant current conservation: $D_\mu j^\mu = 0$

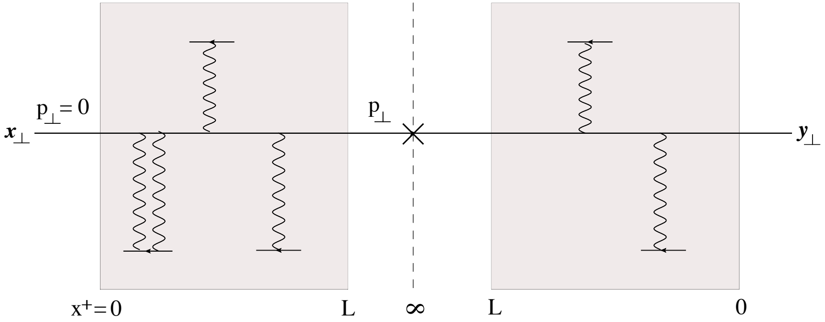
- The fields A_a^- are randomly distributed (since so are their sources)

  $\triangleright$ say, according to the thermal distribution in the case of a QGP

- Cross–sections are obtained after averaging over the background field

Transverse momentum broadening

- Direct amplitude (DA) $\times$ Complex conjugate amplitude (CCA) :



- The $p_\perp$–spectrum of the quark after crossing the medium:

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \frac{1}{(2\pi)^2}\int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}} \langle S_{\boldsymbol{xy}}\rangle\,, \qquad S_{\boldsymbol{xy}} \equiv \frac{1}{N_c}\,\mathrm{tr}\big(V_{\boldsymbol{x}}V_{\boldsymbol{y}}^\dagger\big)$$

▷ sum over the final color indices, average over the initial ones

▷ average over the distribution of the medium field A_a^-

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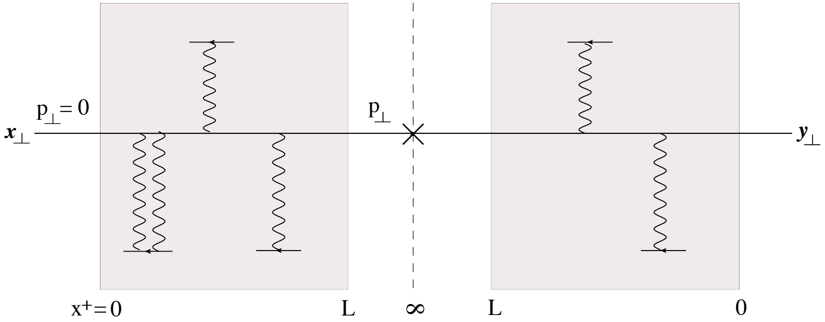

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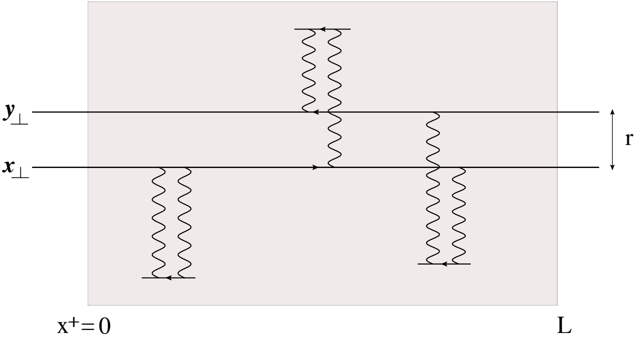
$\triangleright$ check normalization: $\int_{\boldsymbol{p}}(\mathrm{d}N/\mathrm{d}^2\boldsymbol{p}) = 1$ since $S_{\boldsymbol{xy}} = 1$ when $\boldsymbol{x} = \boldsymbol{y}$

Dipole picture

- Formally, $\langle S_{\boldsymbol{xy}} \rangle$ is the average S–matrix for a $q\bar{q}$ color dipole

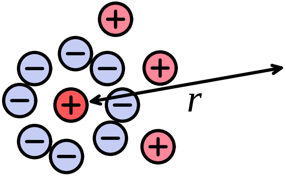


▷ 'the quark at $\boldsymbol{x}$' : the physical quark in the DA

▷ 'the antiquark at $\boldsymbol{y}$' : the physical quark in the CCA

- Quark cross–section $\longleftrightarrow$ dipole amplitude : a useful analogy

- A collection of quasi–independent color charges (quarks and gluons)
  - ▷ e.g. a nearly ideal quark–gluon plasma
- Even at weak coupling, some effects of the interactions are essential
  - ▷ collective phenomena leading to the screening of the gauge interactions



$$V(r) = \frac{\exp(-m_{\text{debye}}\, r)}{r}$$

$$A^0(\mathbf{r}) = \int d^3\mathbf{r}' \frac{e^{-m_D|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\rho(\mathbf{r}') = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{\rho(\mathbf{k})}{\mathbf{k}^2 + m_D^2}$$

- Weakly coupled QGP : $m_D \sim gT$ *(see lectures by Ghiglieri)*
  - ▷ Debye mass acts as an 'infrared' ($k \to 0$) cutoff

Charge & field correlations

- View the scattering process in a boosted Lorentz frame, where the medium is a rapid 'left mover' : $v_z < 0, \ |v_z| \simeq 1$

- The color current density of the medium: $J^\mu_a(x) \simeq \delta^{\mu -}\rho_a(x^+, \boldsymbol{x}_\perp)$

- The gauge field in gauge $A^+ = 0$: $A^\mu_a(x) = \delta^{\mu -} A^-_a(x^+, \boldsymbol{x}_\perp)$

$$A^-_a(x^+, \boldsymbol{x}_\perp) \, = \int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^2}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\, \frac{\rho_a(x^+, \boldsymbol{k}_\perp)}{\boldsymbol{k}^2_\perp + m^2_D}$$

  ▷ local in x^+ due to Lorentz–contraction

  ▷ Coulomb propagator in two (transverse) directions

- The 2–point 'correlation' function of independent color sources:

$$\langle \rho_a(x^+, \boldsymbol{x}_\perp)\, \rho_b(y^+, \boldsymbol{y}_\perp)\rangle \, = \, g^2 n_0\, \delta_{ab}\delta(x^+ - y^+)\, \delta^{(2)}(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$

▷ $n_0 \sim T^3$: quark and gluon densities weighted with color factors

Charge & field correlations

- View the scattering process in a boosted Lorentz frame, where the medium is a rapid 'left mover' : $v_z < 0, \ |v_z| \simeq 1$
- The color current density of the medium: $J_a^\mu(x) \simeq \delta^{\mu -} \rho_a(x^+, \boldsymbol{x}_\perp)$
- The gauge field in gauge $A^+ = 0$: $A_a^\mu(x) = \delta^{\mu -} A_a^-(x^+, \boldsymbol{x}_\perp)$

$$A_a^-(x^+, \boldsymbol{x}_\perp) \, = \int \frac{\mathrm{d}^2 \boldsymbol{k}_\perp}{(2\pi)^2} \, \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp} \, \frac{\rho_a(x^+, \boldsymbol{k}_\perp)}{\boldsymbol{k}_\perp^2 + m_D^2}$$

▷ local in x^+ due to Lorentz–contraction

▷ Coulomb propagator in two (transverse) directions

- The ensuing 2–point correlation function of the gauge fields:

$$\langle A_a^-(x^-, x^+, \boldsymbol{x}_\perp) \, A_b^-(x^-, y^+, \boldsymbol{y}_\perp) \rangle \, = \, g^2 n_0 \, \delta_{ab} \delta(x^+ - y^+) \, \gamma(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)$$

$$\gamma(\boldsymbol{k}_\perp) \, \equiv \, \frac{1}{(\boldsymbol{k}_\perp^2 + m_D^2)^2} \quad : \quad \text{Coulomb propagator squared}$$

▷ 2 gluon exchange with a same medium constituent

The dipole S–matrix (1)

$$\langle S_{\boldsymbol{xy}}\rangle \,=\, \frac{1}{N_c}\left\langle \mathrm{tr}\big(V(\boldsymbol{x})V^{\dagger}(\boldsymbol{y})\big)\right\rangle, \qquad V(\boldsymbol{x}) = \mathrm{T}\,\mathrm{e}^{\mathrm{i}g\int \mathrm{d}x^+\, A_a^-(x^+,\boldsymbol{x})t^a}$$

- Correlations are local in x^+ $\Longrightarrow$ discretize the respective range:

  $\triangleright$ $L = N\epsilon\,,\; x_n^+ = n\epsilon\,,\; A_n^-(\boldsymbol{x}) \equiv A_a^-(x_n^+,\boldsymbol{x})t^a$

$$V_N(\boldsymbol{x}) \,=\, \mathrm{e}^{\mathrm{i}g\epsilon A_N^-(\boldsymbol{x})}\,\mathrm{e}^{\mathrm{i}g\epsilon A_{N-1}^-(\boldsymbol{x})}\,\dots\,\mathrm{e}^{\mathrm{i}g\epsilon A_0^-(\boldsymbol{x})} \,=\, \mathrm{e}^{\mathrm{i}g\epsilon A_N^-(\boldsymbol{x})}\,V_{N-1}(\boldsymbol{x})$$

$$S_N(\boldsymbol{x},\boldsymbol{y}) \,=\, \frac{1}{N_c}\,\mathrm{tr}\Big(\mathrm{e}^{\mathrm{i}g\epsilon A_N^-(\boldsymbol{x})}\,S_{N-1}(\boldsymbol{x},\boldsymbol{y})\,\mathrm{e}^{-\mathrm{i}g\epsilon A_N^-(\boldsymbol{y})}\Big)$$

- The Gaussian correlations in such discretized notations:

$$\langle A_{a,n}^-(\boldsymbol{x})\,A_{b,m}^-(\boldsymbol{y})\rangle \,=\, g^2 n_0\,\delta_{ab}\,\frac{1}{\epsilon}\delta_{nm}\,\gamma(\boldsymbol{x}-\boldsymbol{y})$$

- Correlations at different times factorize from each other !

The dipole S–matrix (2)

- Recurrence formula for $\langle S_n(\boldsymbol{x},\boldsymbol{y})\rangle$:

$$\langle S_n(\boldsymbol{x},\boldsymbol{y})\rangle = \frac{1}{N_c}\left\langle \mathrm{tr}\left[\mathrm{e}^{\mathrm{i}g\epsilon A_n^-(\boldsymbol{x})}\mathrm{e}^{-\mathrm{i}g\epsilon A_n^-(\boldsymbol{y})}\right]\right\rangle \langle S_{n-1}(\boldsymbol{x},\boldsymbol{y})\rangle$$

- Expand up to quadratic order in ϵA_n^-, i.e. to linear order in ϵ

  ▷ the linear terms from the 2 exponentials average with each other

$$\frac{1}{N_c}\left\langle \mathrm{tr}\left(\mathrm{i}g\epsilon A_{n,a}^-(\boldsymbol{x})t^a\right)\left(-\mathrm{i}g\epsilon A_{n,b}^-(\boldsymbol{y})t^b\right)\right\rangle = C_F(g^2\epsilon)(g^2n_0)\gamma(\boldsymbol{x}-\boldsymbol{y})$$

  ▷ the quadratic terms self–average (‘tadpoles’)

$$\frac{(\mathrm{i}g\epsilon)^2}{2}\,\frac{1}{N_c}\left\langle \mathrm{tr}\left(A_{n,a}^-(\boldsymbol{x})t^a\,A_{n,b}^-(\boldsymbol{x})t^b\right)\right\rangle = -C_F\,\frac{g^2\epsilon}{2}\,(g^2n_0)\gamma(0)$$

- Evolution equation for $\langle S_n(\boldsymbol{x},\boldsymbol{y})\rangle$ w.r.t x^+ :

$$\frac{\langle S_n(\boldsymbol{x},\boldsymbol{y})\rangle - \langle S_{n-1}(\boldsymbol{x},\boldsymbol{y})\rangle}{\epsilon} = -g^4C_Fn_0\left[\gamma(0)-\gamma(\boldsymbol{x}-\boldsymbol{y})\right]\langle S_{n-1}(\boldsymbol{x},\boldsymbol{y})\rangle$$

The dipole S–matrix (3)

- Continuum limit $\epsilon \to 0 \Longrightarrow$ equation for $\langle S_t(\boldsymbol{x},\boldsymbol{y})\rangle$ $(t \equiv x^+ \in [0,\ L])$

$$\frac{\partial \langle S_t(\boldsymbol{x},\boldsymbol{y})\rangle}{\partial t} = -g^4 C_F n_0 \big[\gamma(0) - \gamma(\boldsymbol{x}-\boldsymbol{y})\big] \langle S_t(\boldsymbol{x},\boldsymbol{y})\rangle$$

- The solution $\langle S(\boldsymbol{x},\boldsymbol{y})\rangle \equiv \langle S_L(\boldsymbol{x},\boldsymbol{y})\rangle$ is clearly an exponential :

$$\langle S(\boldsymbol{x},\boldsymbol{y})\rangle = \exp\Big\{ - g^4 C_F n_0 L \big[\gamma(0) - \gamma(\boldsymbol{x}-\boldsymbol{y})\big]\Big\}$$

  ▷ $\gamma(0) > \gamma(\boldsymbol{x}-\boldsymbol{y}) \Longrightarrow$ the exponent is positive $\Longrightarrow$ attenuation

  ▷ $\langle S(\boldsymbol{x},\boldsymbol{y})\rangle$ is a function of the dipole size $\boldsymbol{r} = \boldsymbol{x}-\boldsymbol{y}$ (by homogeneity)

- Exponent: amplitude for a single scattering via 2–gluon exchange

  ▷ the dipole is a color–singlet and must remains so after each scattering
  $\Longrightarrow$ a single scattering involves the exchange of two gluons

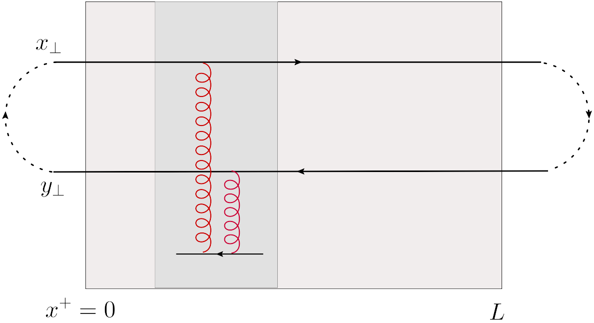
- Independent successive scatterings exponentiate (Glauber series)

Diagrammatic interpretation

- The amplitude for a single scattering:

$$\langle T(\boldsymbol{x},\boldsymbol{y})\rangle_0 = g^4 C_F n_0 L\big[\gamma(0)-\gamma(\boldsymbol{x}-\boldsymbol{y})\big]$$

- The dipole is a color–singlet and must remains so after the scattering $\Longrightarrow$ a single scattering involves the exchange of two gluons



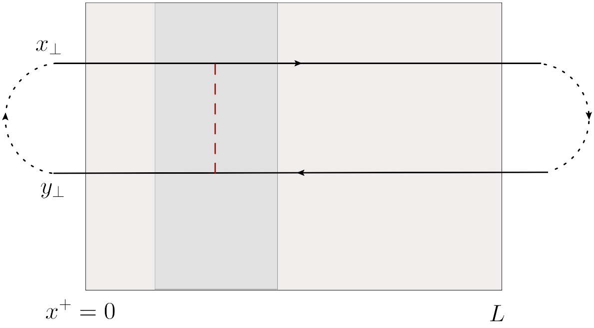
- The medium correlations are Gaussian $\Longrightarrow$ both gluons are exchanged with a same 'color source' from the medium

Diagrammatic interpretation

- The amplitude for a single scattering:

$$\langle T(\boldsymbol{x}, \boldsymbol{y})\rangle_0 = g^4 C_F n_0 L \big[\gamma(0) - \gamma(\boldsymbol{x} - \boldsymbol{y})\big]$$

- The dipole is a color–singlet and must remains so after the scattering $\Longrightarrow$ a single scattering involves the exchange of two gluons



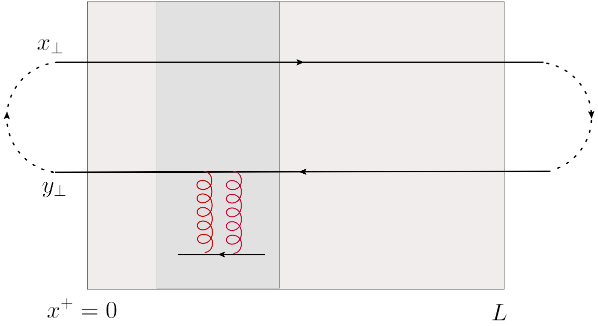
- The medium correlations are local in time (x^+) $\Longrightarrow$ instantaneous exchange between the quark and the antiquark

Diagrammatic interpretation

- The amplitude for a single scattering:

$$\langle T(\boldsymbol{x},\boldsymbol{y})\rangle_0 \,=\, g^4 C_F n_0 L\big[\gamma(0)-\gamma(\boldsymbol{x}-\boldsymbol{y})\big]$$

- The dipole is a color–singlet and must remains so after the scattering $\Longrightarrow$ a single scattering involves the exchange of two gluons



- The two gluons can also be exchanged with a same fermion (q or $\bar{q}$)

Diagrammatic interpretation

- The amplitude for a single scattering:

$$\langle T(\boldsymbol{x}, \boldsymbol{y})\rangle_0 = g^4 C_F n_0 L \big[\gamma(0) - \gamma(\boldsymbol{x}-\boldsymbol{y})\big]$$

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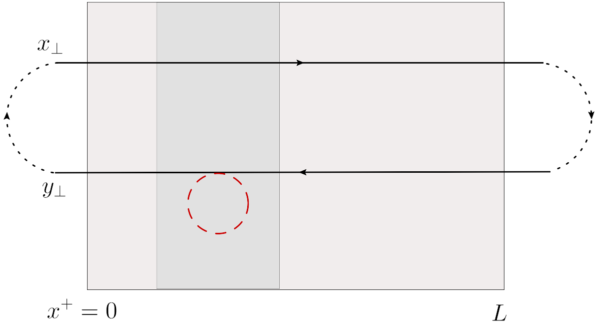

- Effectively, a self–energy tadpole: $\gamma(0)$

The jet quenching parameter (1)

$$\langle S(\boldsymbol{r})\rangle = \exp\left\{ -g^4 C_F n_0 L \int \frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{(\mathbf{k}^2+m_D^2)^2} \left(1-\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\right)\right\}$$

- We are interested in relatively small dipole sizes $r \sim 1/p_\perp \ll 1/m_D$
  - $p_\perp \gg m_D$: the momentum accumulated after many scatterings, each one contributing an amount $\sim m_D$
- The integral is dominated by $k_\perp$ in the range $m_D \ll k_\perp \ll 1/r$
  - this range produces a large logarithm $\ln(1/rm_D) \gg 1$
  - it dominates to leading logarithmic accuracy: up to terms of $\mathcal{O}(1)$
- Within this range one can expand the exponential to quadratic order:

$$1 - \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \simeq -\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r} + \frac{1}{2}\big(\boldsymbol{k}\cdot\boldsymbol{r}\big)^2 \longrightarrow \frac{1}{4}k_\perp^2 r^2$$

The jet quenching parameter (2)

- To leading logarithmic accuracy, the dipole S–matrix reads

$$\langle S(\boldsymbol{r})\rangle \simeq \exp\left\{-\frac{1}{4}\,L\hat{q}(1/r^2)\,\boldsymbol{r}^2\right\}$$

- The jet quenching parameter $\hat{q}$:

$$\hat{q}(1/r^2) \equiv g^4 C_F n_0 \int^{1/r^2} \frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\,\boldsymbol{k}^2\,\frac{1}{(\boldsymbol{k}^2+m_D^2)^2} \simeq 4\pi\alpha_s^2 C_F n_0 \ln\frac{1}{r^2 m_D^2}$$

▷ a property of the medium $(m_D,\, n_0)$ measured on the resolution scale r

- The typical value of r is fixed by multiple scattering

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \frac{1}{(2\pi)^2}\int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}\ \mathrm{e}^{-\frac{1}{4}L\hat{q}(1/r^2)\,\boldsymbol{r}^2}$$

▷ dominated by values $r \sim 1/Q_s$ such that the exponent is of $\mathcal{O}(1)$

The saturation momentum

$$Q_s^2 \;=\; L\hat{q}(Q_s^2) \;=\; 4\pi\alpha_s^2 C_F n_0 L \ln\frac{Q_s^2}{m_D^2} \;\propto\; L\ln L$$

- For the dipole : the borderline of multiple scattering
- For the quark: the typical transverse momentum broadening

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} \;\simeq\; \frac{1}{(2\pi)^2}\int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}\;\mathrm{e}^{-\frac{1}{4}Q_s^2\,\boldsymbol{r}^2} \;=\; \frac{1}{\pi Q_s^2}\,\mathrm{e}^{-\boldsymbol{p}^2/Q_s^2}$$

- Gaussian $\Longrightarrow$ a random walk in $p_\perp$: $\langle p_\perp^2\rangle = Q_s^2 = \hat{q}(Q_s^2)L$
- The physical jet quenching parameter : $\hat{q}(Q_s^2) \propto \ln L$

  $\triangleright$ weak dependence upon the medium size L

- The physics of $p_\perp$–broadening is mildly non–local in time

  $\triangleright$ it 'knows' about the overall size of the medium

The tail of the distribution at high $p_\perp$

- So far, we implicitly assumed that $p_\perp$ is not much larger than Q_s

  $\triangleright$ the typical situation: $p_\perp$ gets accumulated via rescattering in the medium

- When $p_\perp \gg Q_s$, the Fourier transform is cut off by the complex exponential $\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}$ already for small dipole sizes $r_\perp \ll 1/Q_s$

- The S–matrix can be evaluated in the single scattering approximation:

$$\langle S(\boldsymbol{r})\rangle \simeq 1 - \frac{1}{4}\,L\hat{q}(1/r^2)\,\boldsymbol{r}^2$$

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} \simeq \frac{\alpha_s^2 C_F}{4\pi} n_0 L \int_{\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}\,(-r^2)\ln\frac{1}{r^2 m_D^2} = \frac{4\alpha_s^2 C_F n_0 L}{\boldsymbol{p}^4}$$

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  ▷ N.B. the logarithmic dependence of $\hat{q}(1/r^2)$ is now essential

- The high–$p_\perp$ tail is generated via rare but very hard collisions